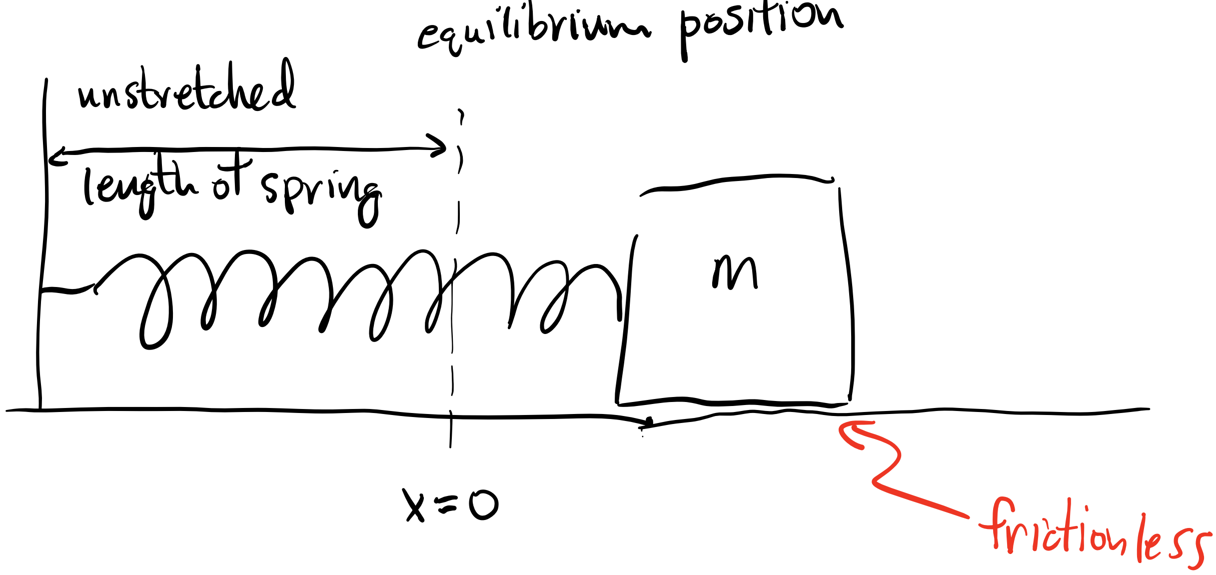
PHYS 121 Jan. 10, 2024

To do:
- complete survey by Jan. 15 @ 23:59
- complete HW1 on PL by Jan. 17 @ 23:59
- complete HW2 on PL by Jan. 19 @ 23:59

Today: Some review of PHYS 111
- Eq'ns of motion
- Free-body diagrams
- Mass on a spring
- Rotational Motion
- Pendulum → Labs #1 & 2.

Mass on Spring



Spring force: Hooke's Law $\vec{F}_s = -k\vec{x}$

Newton's 2nd Law in x-dir'n

$$F_{net,x} = m a_x = -kx \quad \text{(Eq'n motion)}$$

Can solve for the position of the mass x as a fcn of time.

$$m a_x = -kx \qquad a_x = \frac{dv_x}{dt}$$

$$m \frac{dv_x}{dt} = -kx \qquad v_x = \frac{dx}{dt}$$

$$m \frac{d}{dt}\left(\frac{dx}{dt}\right) = -kx$$

$$\therefore\ m \frac{d^2x}{dt^2} = -kx$$

$$\text{or} \quad \frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad (1)$$

To find $x(t)$, require a fcn that after two derivatives w.r.t. time gives minus the orignal fcn.

Try a solution of $x = \underset{\text{amplitude}}{A}\cos(\underset{\text{angular freq.}}{\omega} t)$

$$\frac{dx}{dt} = -\omega A \sin(\omega t)$$

$$\frac{d^2x}{dt^2} = -\omega^2 \underbrace{A\cos\omega t}_{x}$$

$$\therefore \frac{d^2x}{dt^2} = -\omega^2 x \quad (2)$$

If $x = A\cos(\omega t)$ is a valid sol'n, require

(1) = (2)

$$\cancel{-}\frac{k}{m}\cancel{x} = \cancel{-}\omega^2\cancel{x}$$

$$\therefore \boxed{\omega = \sqrt{\frac{k}{m}}}$$ angular freq. of mass on a spring.

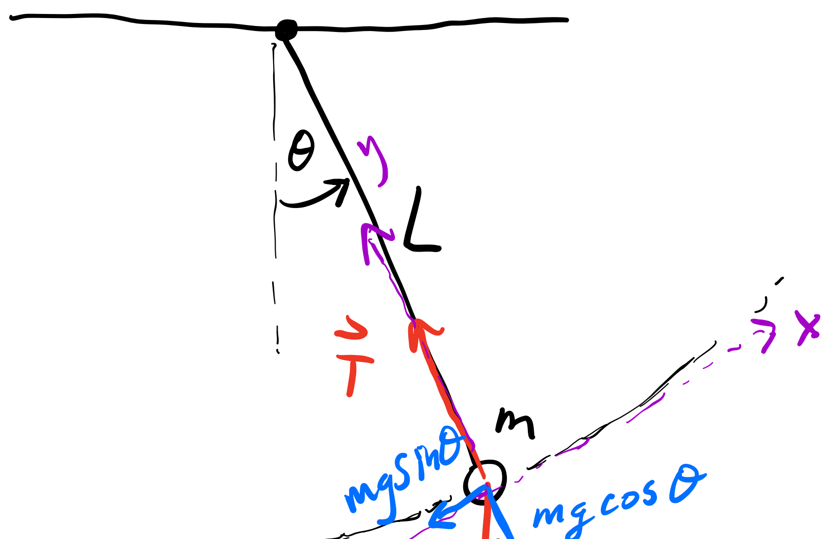
Recall angular freq, $\omega = 2\pi f$
where f is the no. of cycles per second.

Period $T = \frac{1}{f}$ & is the time it takes to complete on full cycle or osc. of the motion.

$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$\therefore \text{Period } \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

<u>Pendulum</u>: Try to relate the prob. of an osc. pendulum to the mass on spring.



Free-body diagram (FBD) for pendum mass.

Decompose forces in x & y components.

2nd Law in y-dir'n:

b/c no motion along y-dir'n.

$$ma_y = T - mg\cos\theta = 0$$

$$\boxed{\therefore\ T = mg\cos\theta}$$

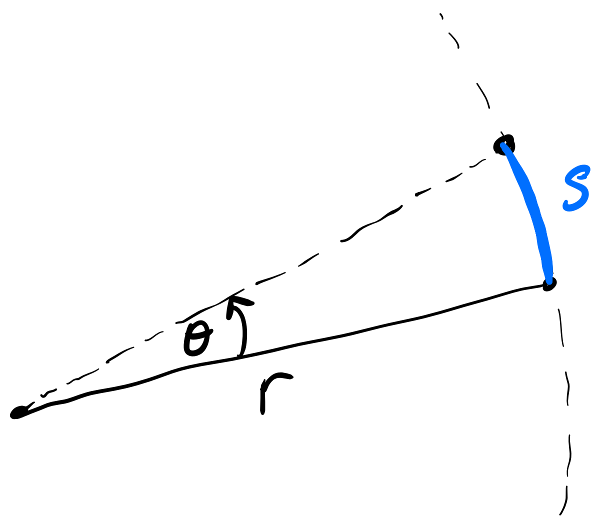
2nd Law in x-dir'n.

$$\cancel{m}a_x = -\cancel{m}g\sin\theta$$

$$a_x = \frac{d^2x}{dt^2} = -g\sin\theta \qquad (3)$$

Note quite the same as the mass on a spring.
Let's see if we can make some manipulations.

Aside: Rotational Motion.



$$s = r\theta$$

$$v = \frac{ds}{dt}$$

$$= \frac{d}{dt}(r\theta)$$

for circular motion
r is const.

$$\boxed{\therefore\ v = r\frac{d\theta}{dt}}$$

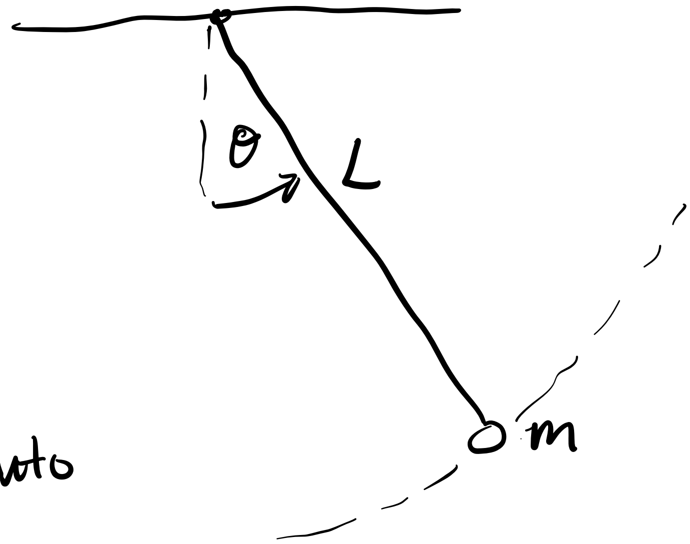
Also know $a = \frac{dv}{dt} = \frac{d}{dt}\left[r\frac{d\theta}{dt}\right]$

$$\boxed{\therefore\ a = r\frac{d^2\theta}{dt^2}}$$

Summary

Linear Motion	Circular Motion
x	$r\theta$
$v = \frac{dx}{dt}$	$r\frac{d\theta}{dt}$
$a = \frac{d^2x}{dt^2}$	$r\frac{d^2\theta}{dt^2}$

Back to pendum:



$\therefore a_x = L \dfrac{d^2\theta}{dt^2}$

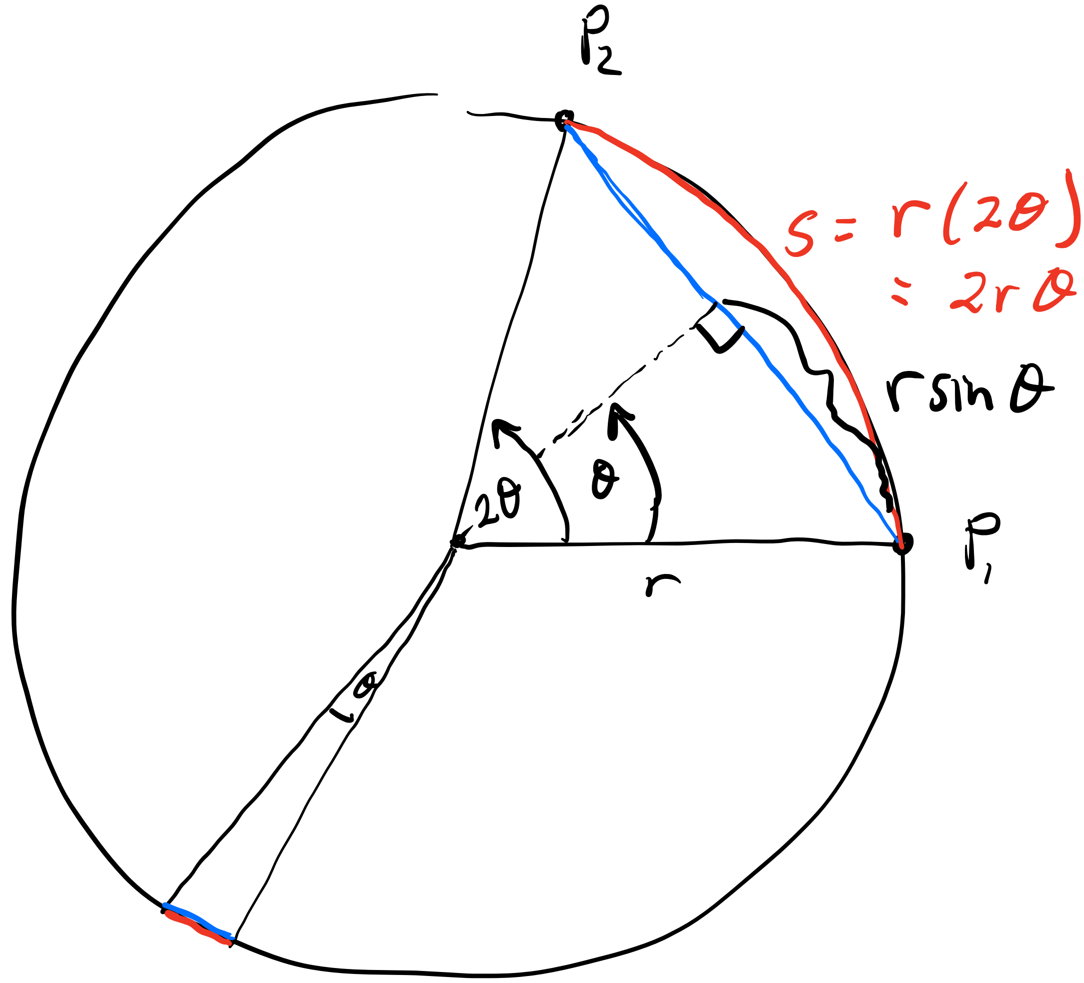
sub this result into Eq. 3

$$L \frac{d^2\theta}{dt^2} = -g \sin\theta$$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta}$$

Pendulum eq'n of motion

c.t. $\underbrace{\dfrac{d^2x}{dt^2} = -\dfrac{k}{m}x}_{\text{mass on a spring.}}$



Know red arc s is longer than blue line d

$s = 2r\theta$

$d = 2r\sin\theta$

$s > d.$