PHYS 121 Jan. 19, 2024

To do: ✓ Complete HW2 by 23:59 tonight.

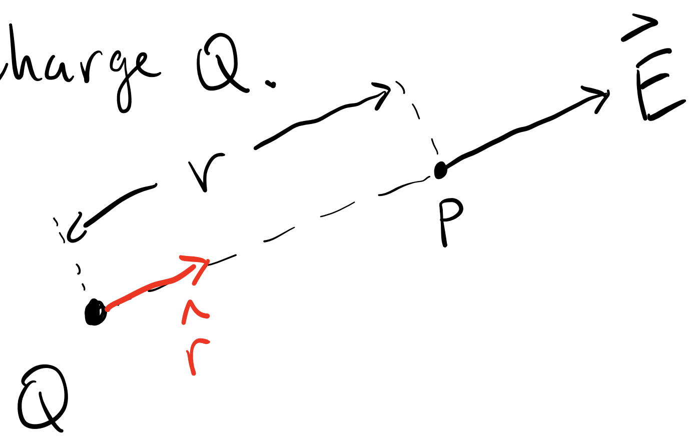
✓ Labs & Tutorials start the week of Jan. 22

✓ Lab #0 next week ⇒ no pre-lab.

Last Time:

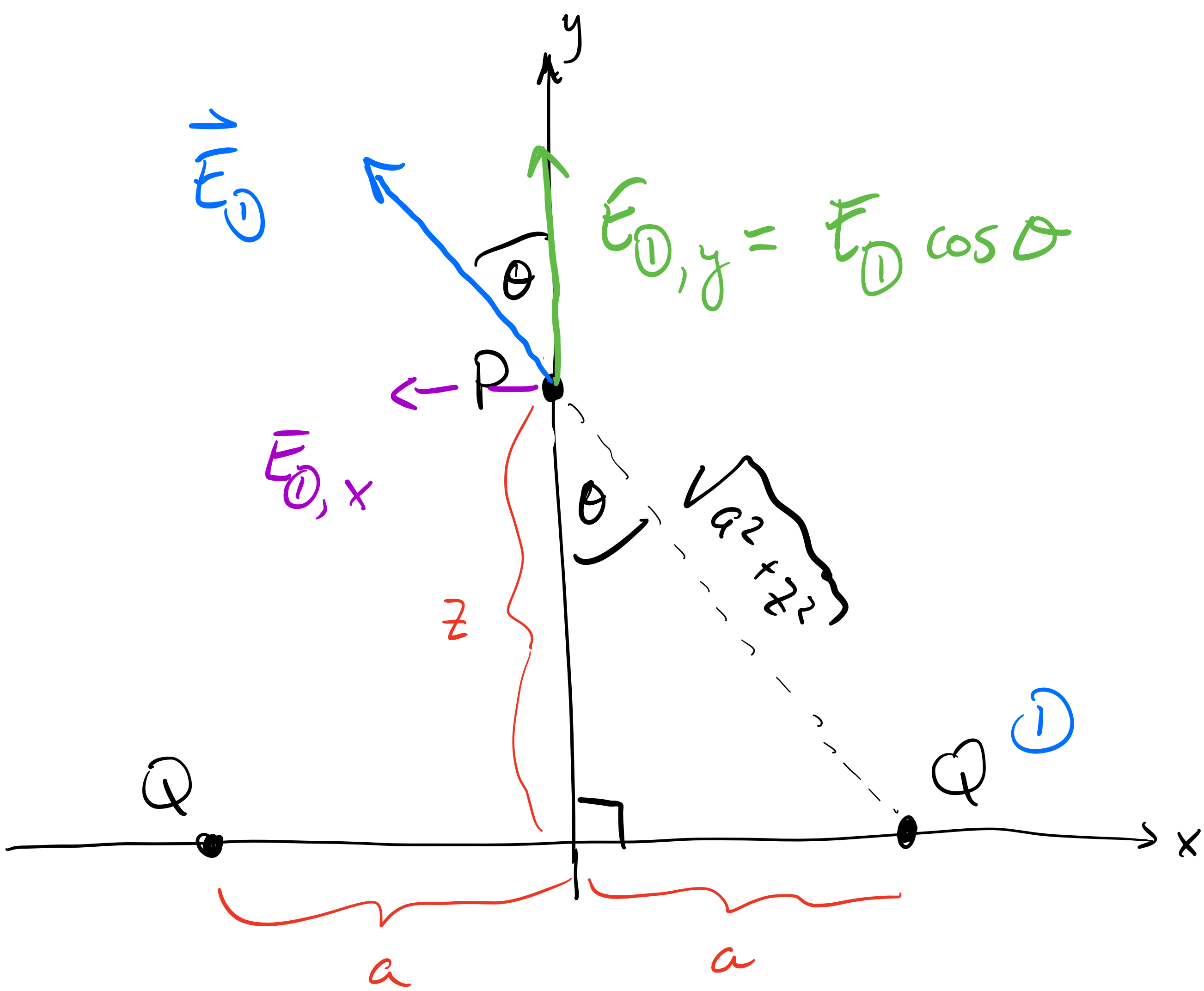
- Force on charge q due to electric field $\vec{E}$: $\boxed{\vec{F} = q\vec{E}}$

- Electric field at P due to point charge Q.



$$\boxed{\vec{E} = \frac{k_e Q}{r^2}\hat{r}}$$

Example: Find the net electric field at pt. P.



The two charges Q are identical & positive.

- individually find the contribution of each charge Q to the Electric field at P.
  $\Rightarrow$ Add the contributions as vectors.

- look for simplifying symmetrie in the geometry that will make the problem easier to solve.

simplifying symmetry

For this problem, $|\vec{E}_{①}| = |\vec{E}_{②}|$ but the x-components are in opposite directions & will cancel exactly.

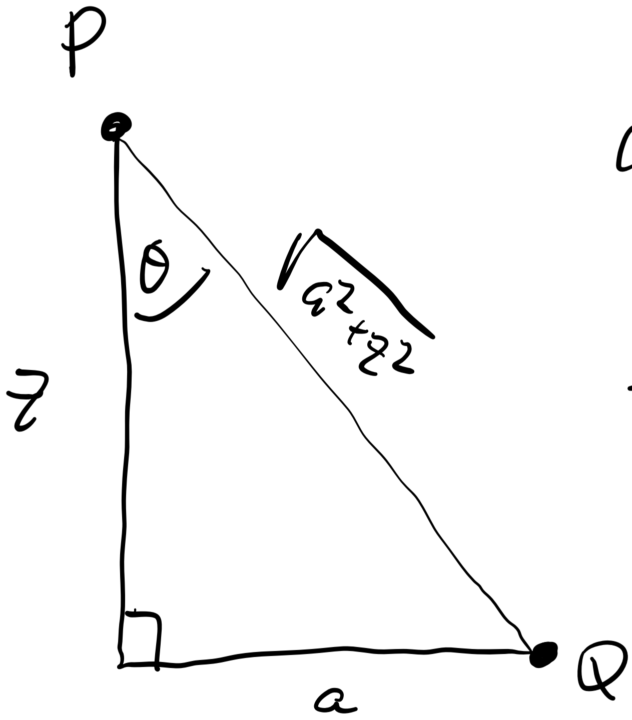
The y-components will be equal & in same dir'n & therefore simply add.

$$E_{net,x} = E_{②,x} - E_{①,x} = 0$$

$$E_{net,y} = E_{①,y} + E_{②,y} = 2E_{①,y}$$

$$= 2E_{①} \cos\theta$$

Know $E_{①} = \boxed{\dfrac{k_e Q}{(a^2+z^2)}}$ for pt. charge.

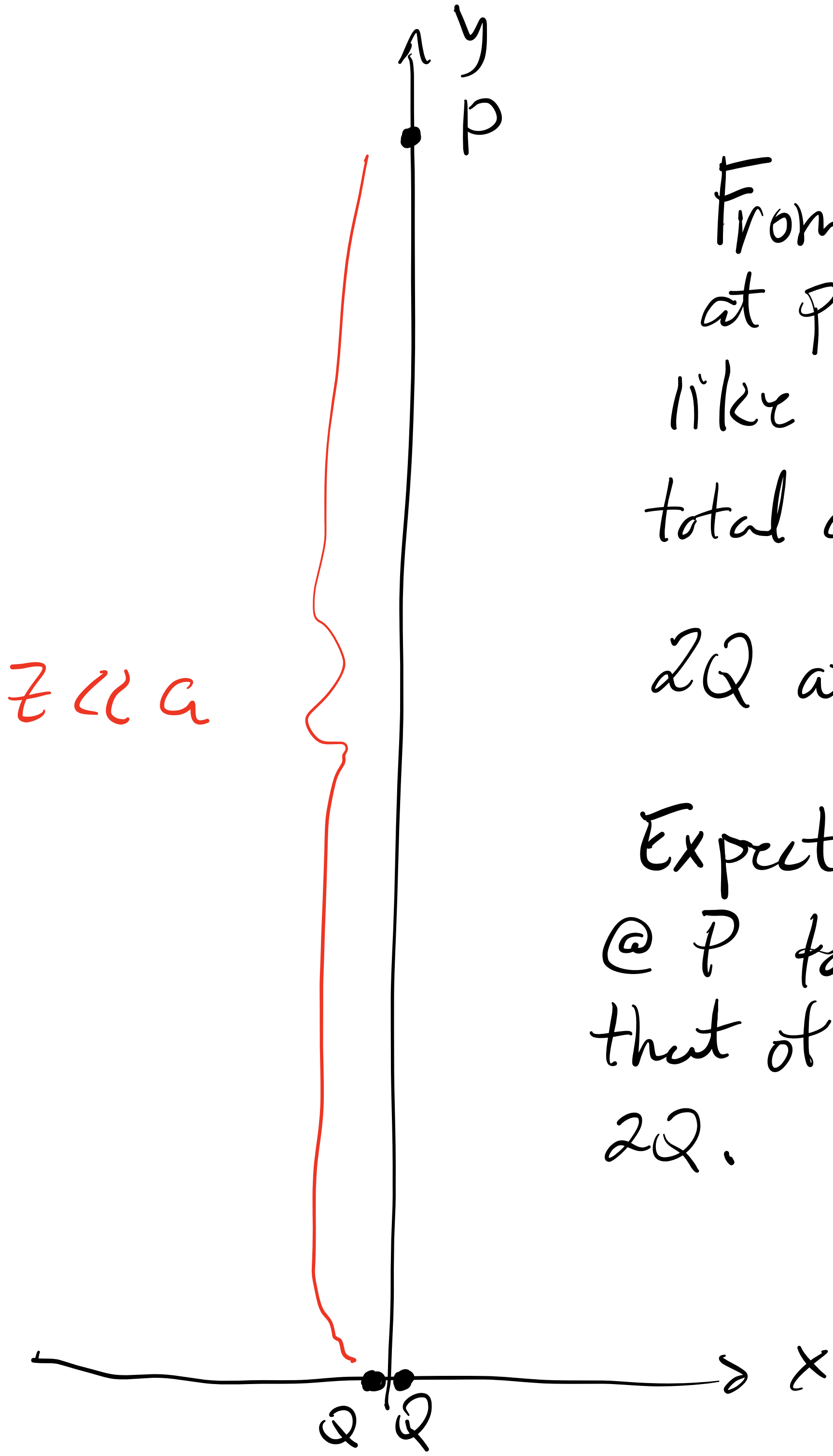


$$\cos\theta = \frac{adj}{hyp} = \boxed{\frac{z}{\sqrt{a^2+z^2}}}$$

$$\therefore E_{net,y} = 2\,\frac{k_e Q}{(a^2+z^2)}\,\frac{z}{\sqrt{a^2+z^2}} = \frac{2k_e Q z}{(a^2+z^2)^{3/2}}$$

$$\boxed{\vec{E}_{net} = 0\,\hat{i} + \frac{2k_e Q z}{(a^2+z^2)^{3/2}}\,\hat{j}}$$ 1

Consider what happens when we make $z \ll a$.



From the position at pt. P, it looks like we have to total charge of

2Q at the origin.

Expect Electric field @ P to look like that of a pt. charge 2Q.

$$\boxed{E \approx \frac{k_e (2Q)}{z^2}}$$

(2)

Expect that Eq. 1 should become equivalent to Eq. 2 in the limit $z \gg a$.

Eq. 1 : $$\frac{k_e 2Q z}{(z^2+a^2)^{3/2}}$$

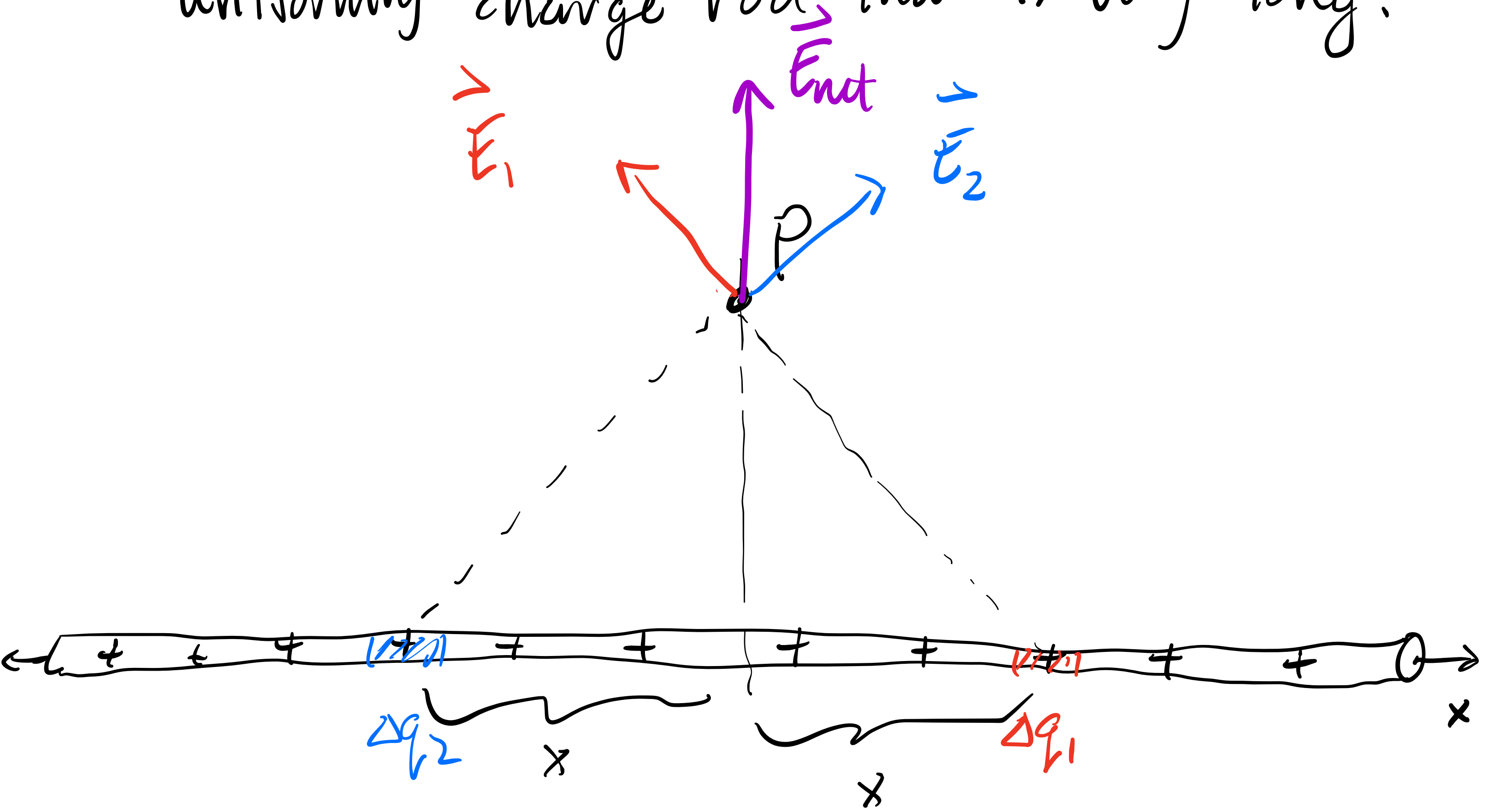
If $z \gg a$, then $z^2+a^2 \approx z^2$

$$(z^2+a^2)^{3/2} \approx (z^2)^{3/2} \approx z^3$$

$$E_{net,y} \approx \frac{k_e 2Q \cancel{z}}{z^{\cancel{3}\,2}} = \frac{k_e (2Q)}{z^2}$$

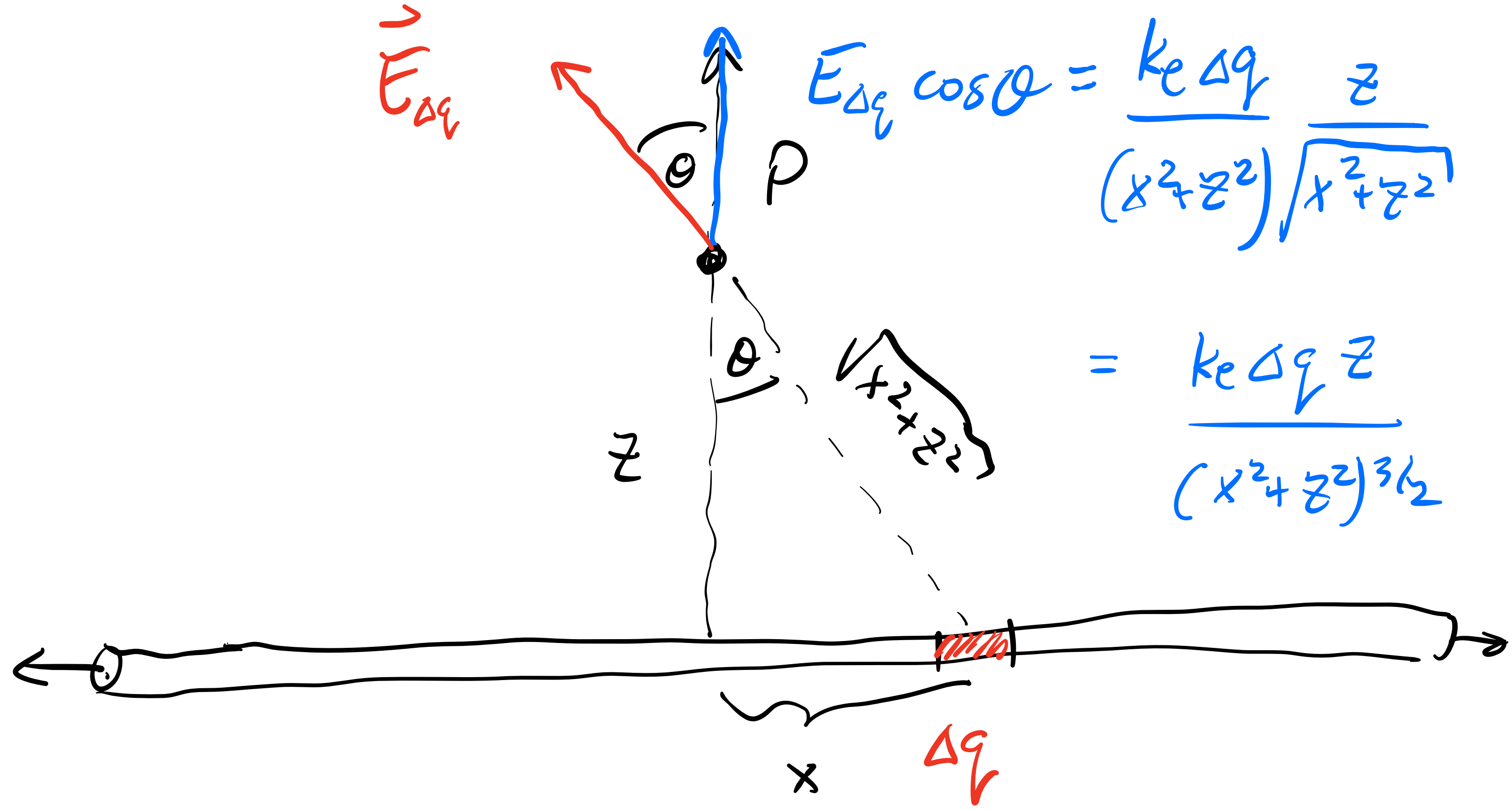
as expected.

Calculate the electric field due to a uniformly-charge rod that is very long.



By symmetry, the net electric field at P must be $\perp$ (or pointing away from) the charged rod.

To find the net electric field at P due to charged rod, add up all of the vertical compenents from the elements of charge Δq that make up the rod.



$$E_{\Delta q}\cos\theta = \frac{k_e \Delta q}{(x^2+z^2)}\,\frac{z}{\sqrt{x^2+z^2}}$$

$$= \frac{k_e \Delta q\, z}{(x^2+z^2)^{3/2}}$$

The net electric due to entire rod is given by the sum over all the Δq's that a need to make up the rod.

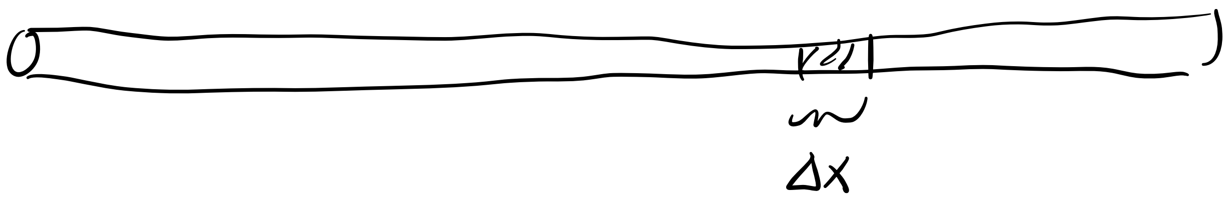
$$E_{net,y} = \sum_{\text{all } \Delta q\text{'s}} \frac{k_e \boxed{\Delta q_i}\, z}{(x_i^2+z^2)^{3/2}}$$

Assume that in a section of this rod that is length L, the total is Q.

Then, we can define a linear charge density (charge per unit length) of

$$\lambda = \frac{Q}{L}$$

Greek letter "lambda"



In a section of rod of length Δx, we have a charge

$$\boxed{\Delta q = \lambda \Delta x}$$

$$\therefore \quad E_{net,y} = \sum_{\substack{i \\ (\text{all } \Delta q\text{'s})}} \frac{k_e \lambda \Delta x \, z}{(x_i^2 + z^2)^{3/2}}$$

This expression can be evaluated as an integral:

$$E_{net,y} = \int_{-\infty}^{\infty} \frac{k_e \lambda z}{(x^2+z^2)^{3/2}}\, dx$$

The result is:

$$\boxed{E_{net,y} = \frac{2k_e\lambda}{z}}$$

using $k_e = \dfrac{1}{4\pi\varepsilon_0}$

$$\boxed{E_{net,y} = \frac{\lambda}{2\pi\varepsilon_0 z}}$$

The electric field a dist. z away from a long uniformly charged rod.