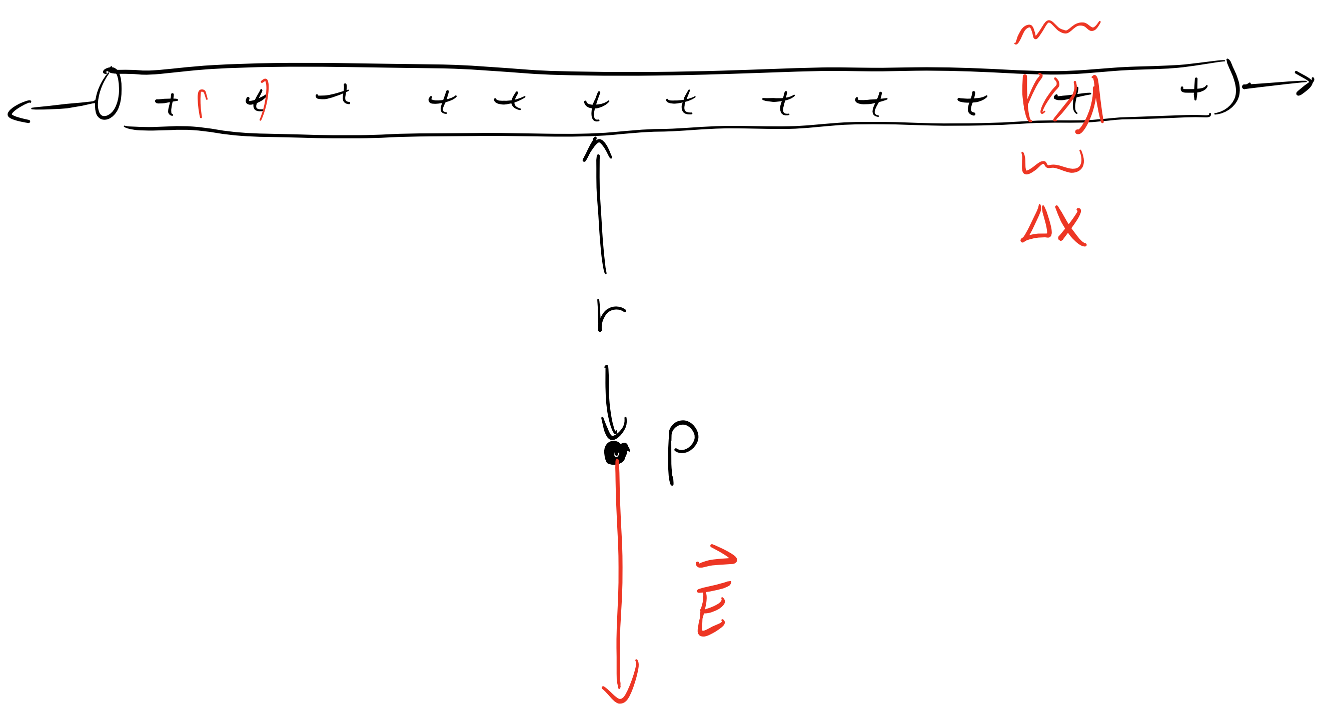
PHYS 121 Jan. 22, 2024

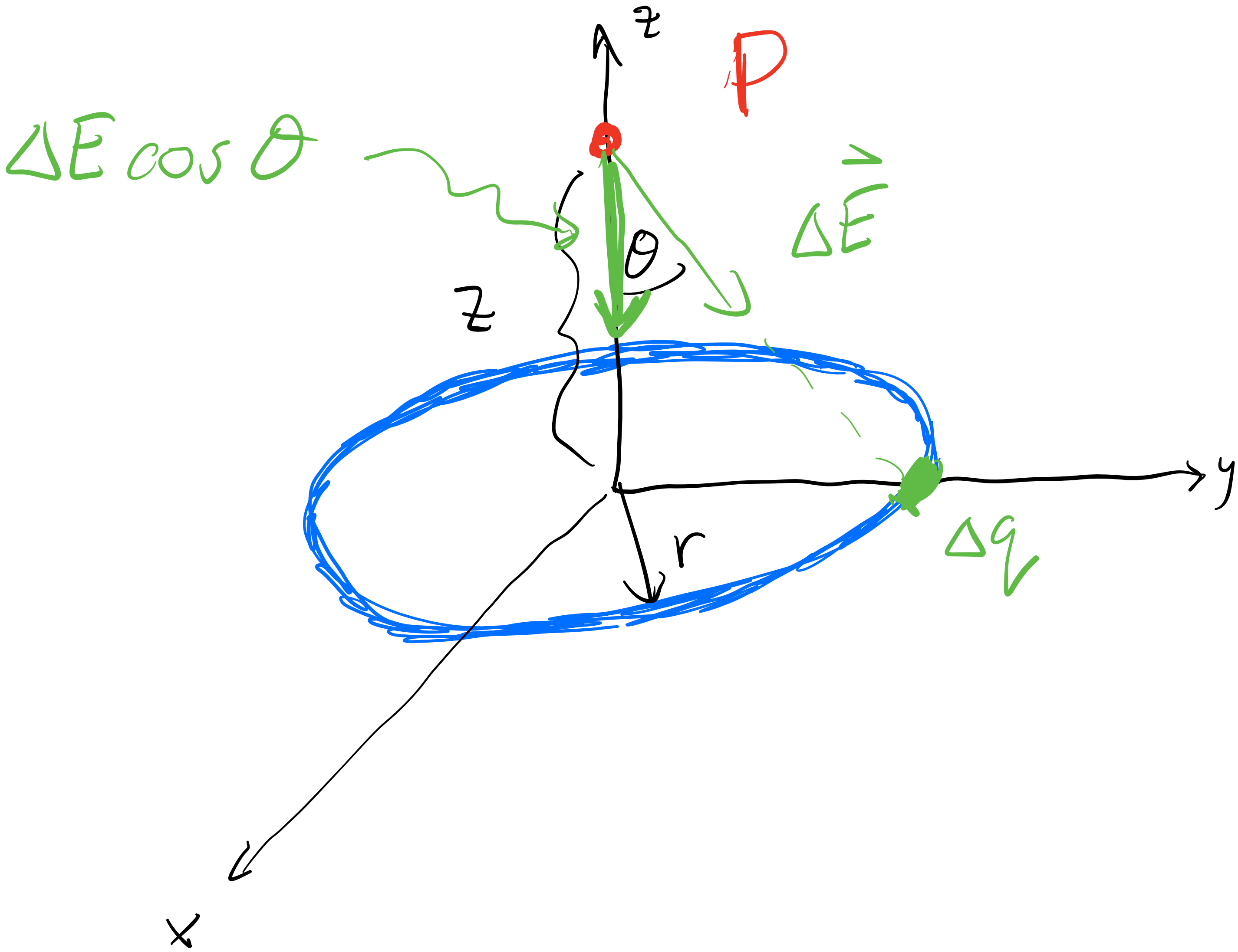
<u>To do</u>: - Complete HW3 by 23:59 on Friday
- Labs & Tutorials start this week.
- No Pre-Lab for Lab #0, but complete Lab #1 Pre-Lab before Lab #1 next week.

<u>Last Time</u>: Electric field due to a line of charge w/ linear charge density $\lambda = \frac{Q}{L}$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

$\Delta q = \lambda \Delta x$



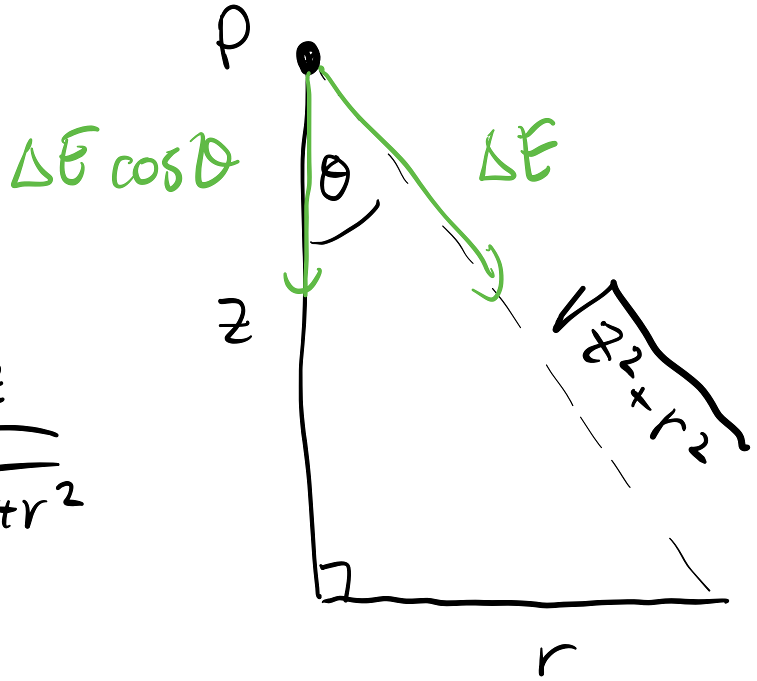


Find the $\vec{E}$-field at a point along the z-axis due to uniformly-charged ring.

Assume that ring has a negative charge $Q_{ring} < 0$.

Charge per unit length $\lambda = \dfrac{Q_{ring}}{2\pi r}$

By symmetry, the net electric field will point along $-z$-axis.



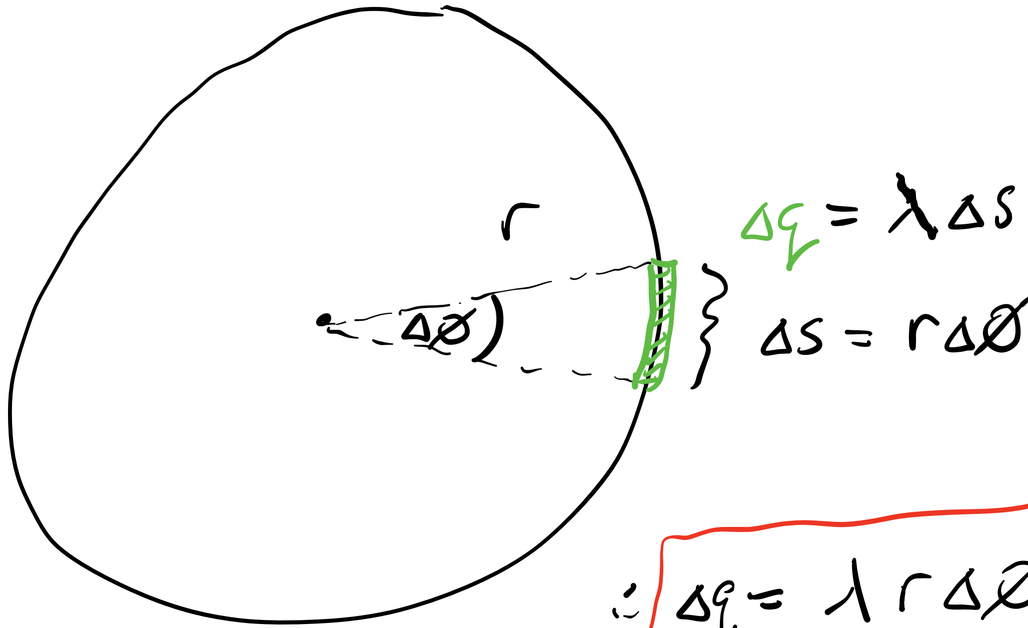
$$\cos\theta = \frac{z}{\sqrt{z^2+r^2}}$$

$$\Delta E_z = \Delta E \cos\theta = \underbrace{\left(\frac{k_e \Delta q}{z^2+r^2}\right)}_{\Delta E} \underbrace{\frac{z}{\sqrt{z^2+r^2}}}_{\cos\theta}$$

$$\Delta E_z = \frac{k_e \Delta q\, z}{(z^2+r^2)^{3/2}}$$

$$E_{net} = \sum_{\substack{\text{point charges} \\ \text{on ring}}} \Delta E_z$$

Top view of ring:



$\therefore \boxed{\Delta q = \lambda r \Delta \varnothing}$

$$E_{net} = \sum_{\substack{\text{all } \Delta q\text{'s} \\ \text{on ring}}} \frac{k_e \lambda r \Delta\varnothing\, z}{(z^2+r^2)^{3/2}}$$

For small $\Delta\varnothing$, sum can be expressed as an integral

$$E_{net} = \int_0^{2\pi} \frac{k_e \lambda r z}{(r^2+z^2)^{3/2}}\, d\varnothing$$

take constants (r, z) outside integral

$$E_{net} = \frac{k_e \lambda r z}{(r^2+z^2)^{3/2}} \underbrace{\int_0^{2\pi} d\phi}_{2\pi}$$

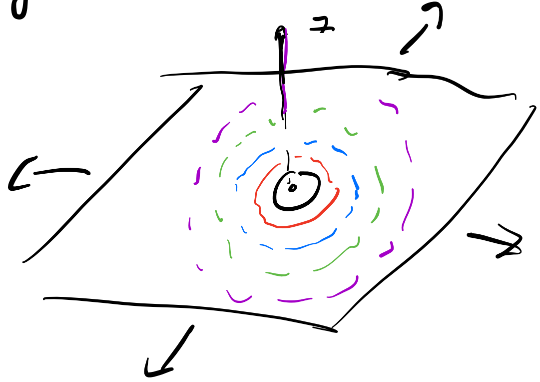
$$\boxed{\therefore\ E_{net} = \frac{k_e (2\pi r \lambda) z}{(z^2+r^2)^{3/2}} = \frac{k_e Q_{ring}\, z}{(z^2+r^2)^{3/2}}}$$

recall $\lambda = \frac{Q_{ring}}{2\pi r}$

$\therefore\ 2\pi r \lambda = Q_{ring}$

Aside: Charged sheet

optional online notes if interested.



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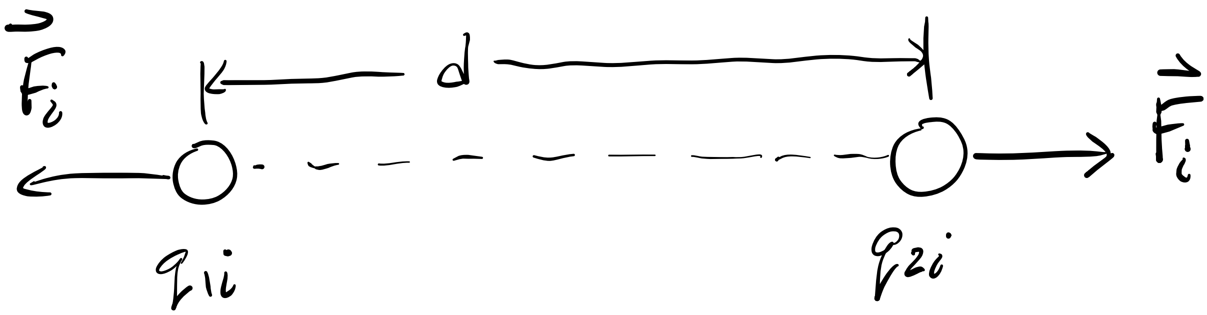
Two small, identical conducting spheres repel each other with a force of $0.5\ \mathrm{N}$ when they are $0.27\ \mathrm{m}$ apart. After a conducting wire is connected between the spheres and then removed, they repel each other with a force of $0.6\ \mathrm{N}$ (the spheres are still separated by $0.27\ \mathrm{m}$).

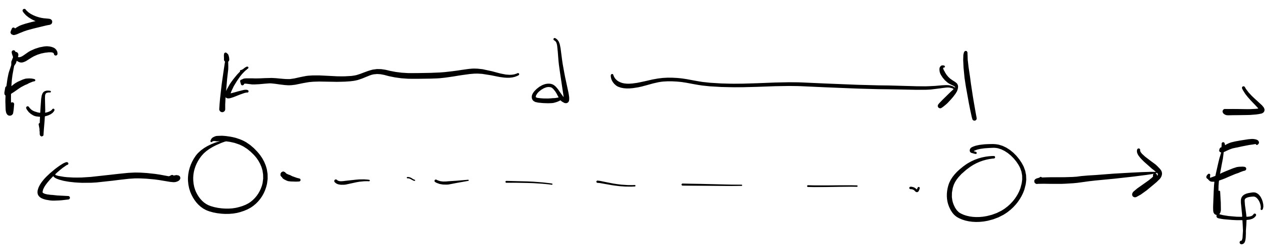
What is the original charge on each sphere? Assume that q_1 is positive and $q_1 > q_2$, where q_1 represents the initial charge on sphere 1 and q_2 is the initial charge on sphere 2.

Initial:



$$\boxed{F_i = \frac{k_e\, q_{1i}\, q_{2i}}{d^2}} \quad (1)$$

Final:



$q_{1f} = q$ $\qquad$ $q_{2f} = q$

$$\boxed{F_f = \frac{k_e\, q^2}{d^2}} \quad (2)$$

Total charge of system remains fixed.

$$q_{1i} + q_{2i} = 2q$$

$$\boxed{\therefore q_{2i} = 2q - q_{1i}} \quad (3)$$

Steps:

(i) use (2) to find q.

(ii) sub (3) into (1) to eliminate q_{2i}.

(iii) From (ii) end up w/ quadratic eq'n for q_{1i}

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$