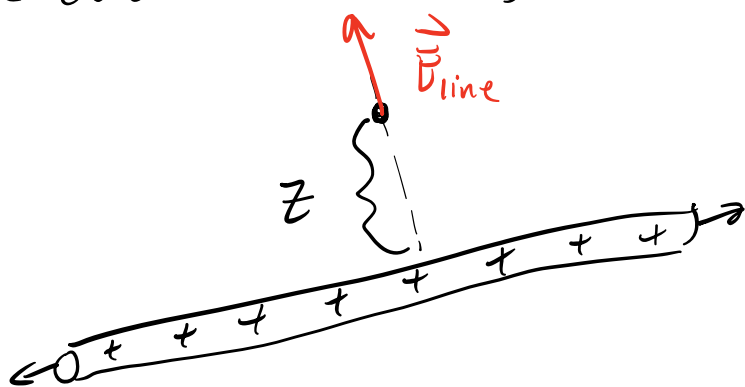


- To do:
- Complete HW3 by 23:59 on Friday
 - Labs & Tutorials start this week.
 - No Pre-Lab for Lab #0, but complete Lab #1 Pre-Lab before Lab #1 next week.

Last Time: Electric fields due to continuous charge distributions:

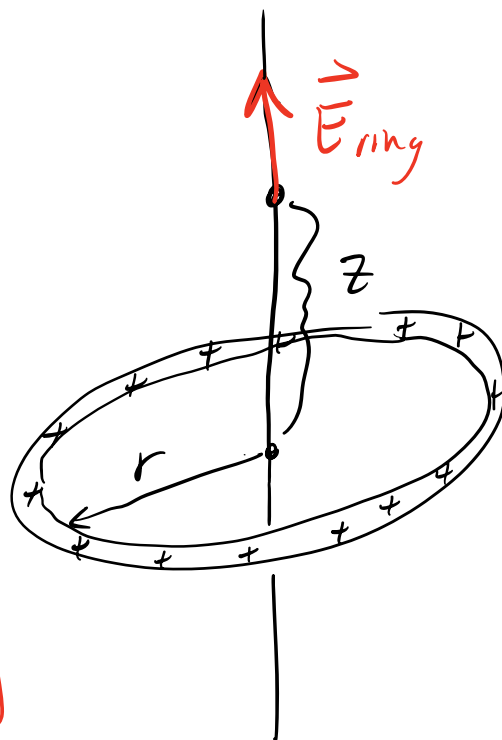
Line of charge

$$E_{\text{line}} = \frac{\lambda}{2\pi\epsilon_0 r}$$



Ring of Charge

$$E_{\text{ring}} = \frac{k_e Q_{\text{ring}} z}{(z^2 + r^2)^{3/2}}$$



$$Q_{\text{ring}} = \lambda (2\pi r)$$

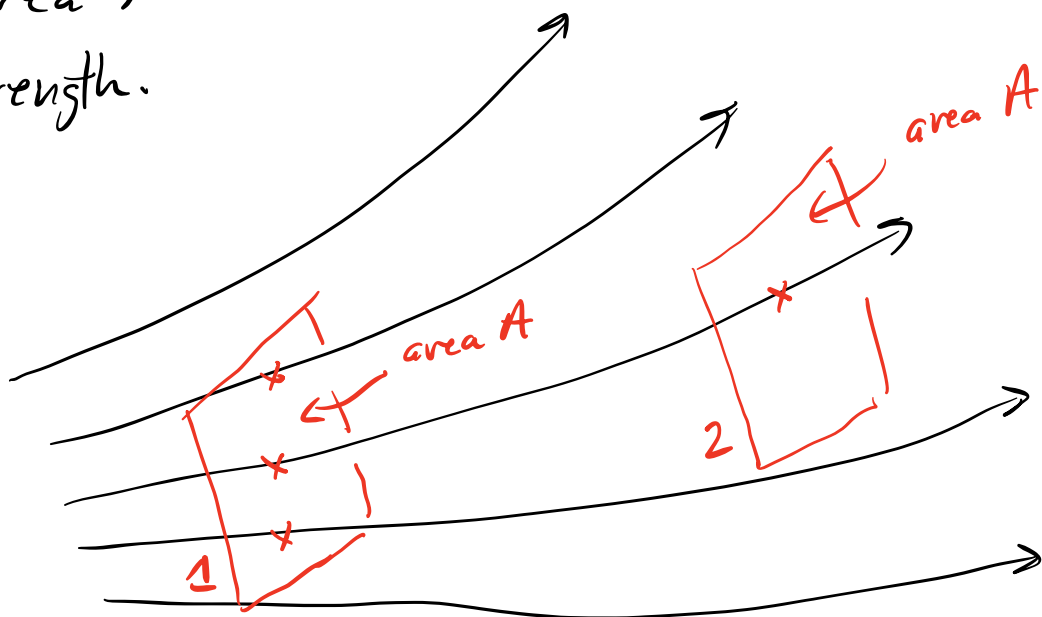
Today: Electric Flux Φ ← greek letter capital Φ .
OSUPv2 chapter 6.

- Why do we care about flux?

▣ The electric flux will help us calculate the electric field due to various charge distributions
 \Rightarrow Gauss's Law

▣ We see later that changing magnetic flux induces voltages
 \Rightarrow Faraday's Law

Recall that the density of electric field lines ($\frac{\# \text{lines}}{\text{area}}$) is proportional to the electric field strength.



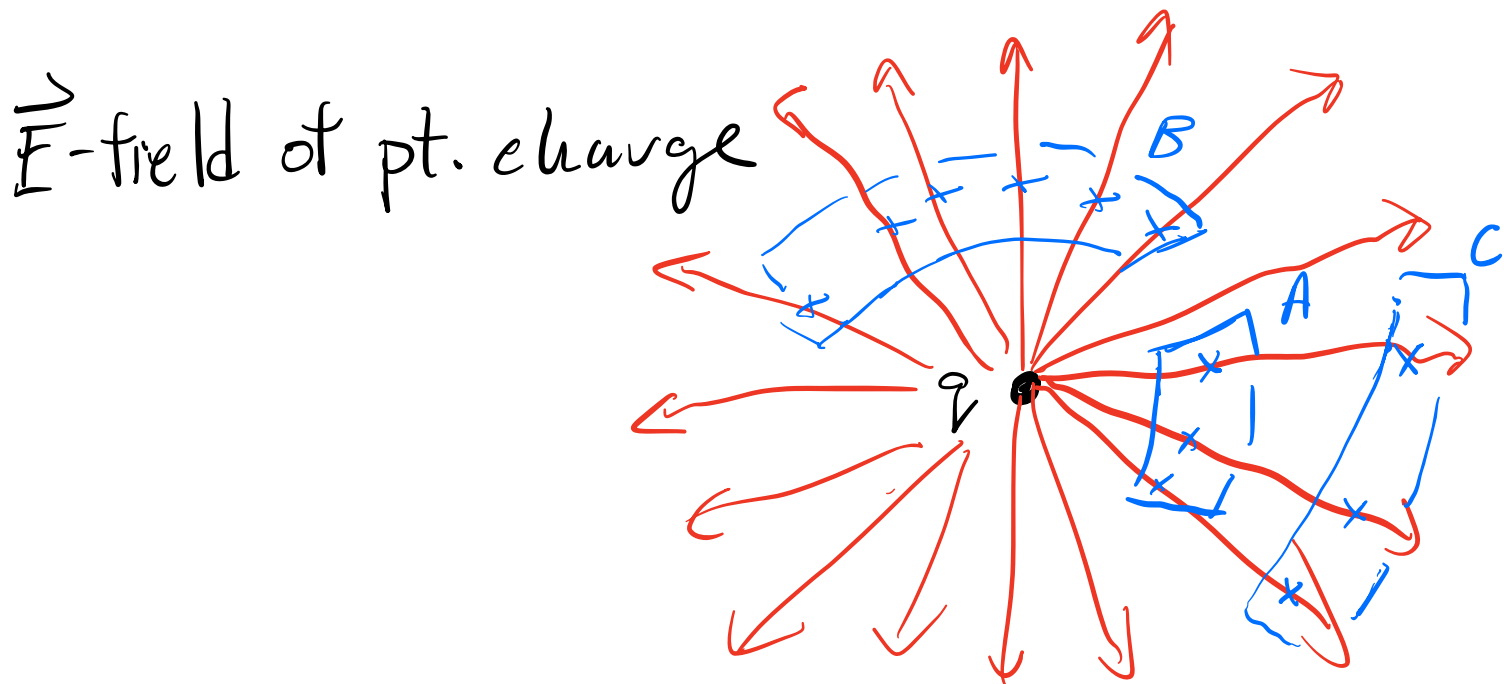
In the example above, three times the no. of \vec{E} -field lines through area @ pos. 1 that at pos. 2.

$$\therefore \frac{|\vec{E}_2|}{|\vec{E}_1|} = \frac{1}{3}$$

The electric flux Φ is defined to be the electric field strength times area of surface.

Since $E \propto \frac{\# \text{ lines}}{\text{area}}$ $\Phi \propto E \cdot \text{area}$

$$\Rightarrow \Phi \propto \left(\frac{\# \text{ lines}}{\text{area}} \right) \text{area} = \# \text{ lines.}$$



Electric Field

$$E_A = E_B \text{ (same dist from } q \text{)}$$

$$E_A = E_B > E_C$$

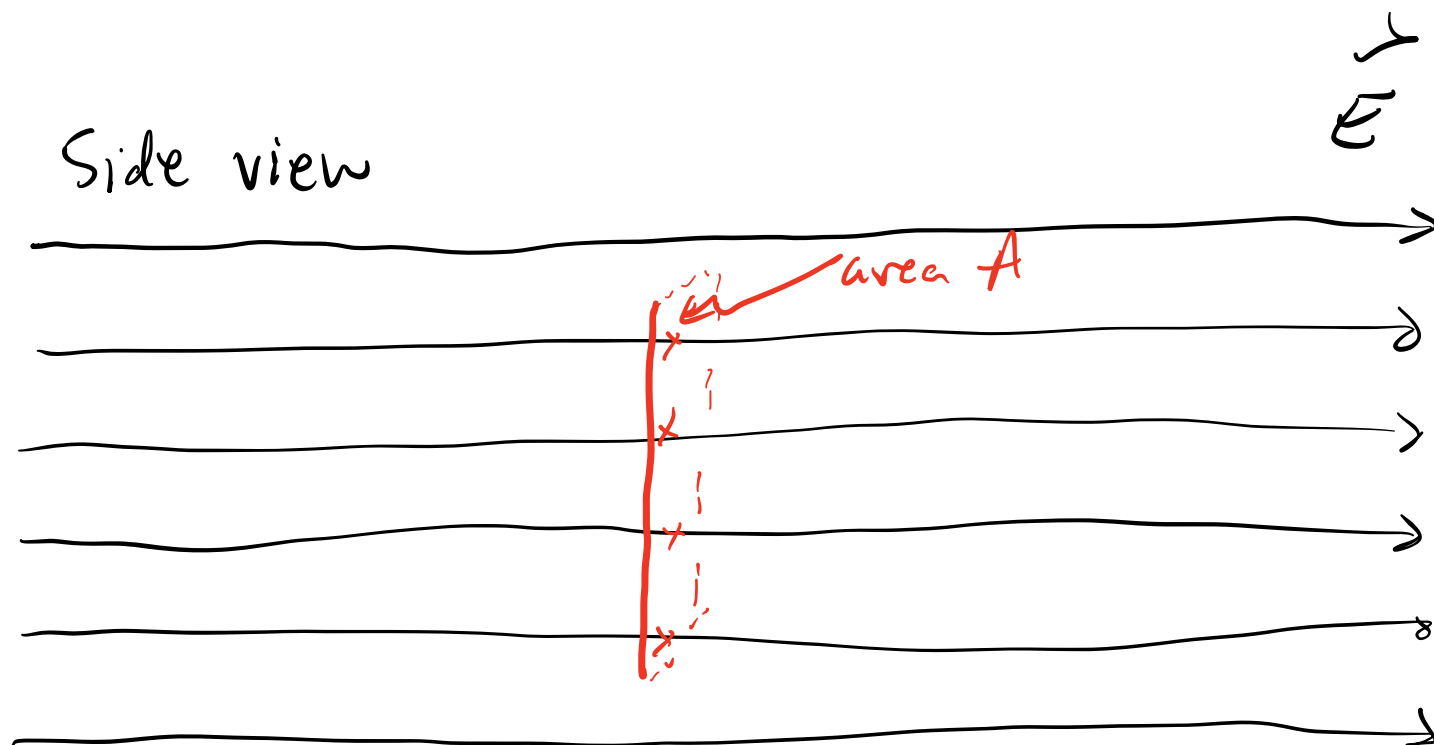
Electric flux

$$\Phi_A < \Phi_B \text{ (# lines)}$$

$$\Phi_A = \Phi_C < \Phi_B$$

Imagine a uniform electric field & a surface \perp to \vec{E} .

Side view

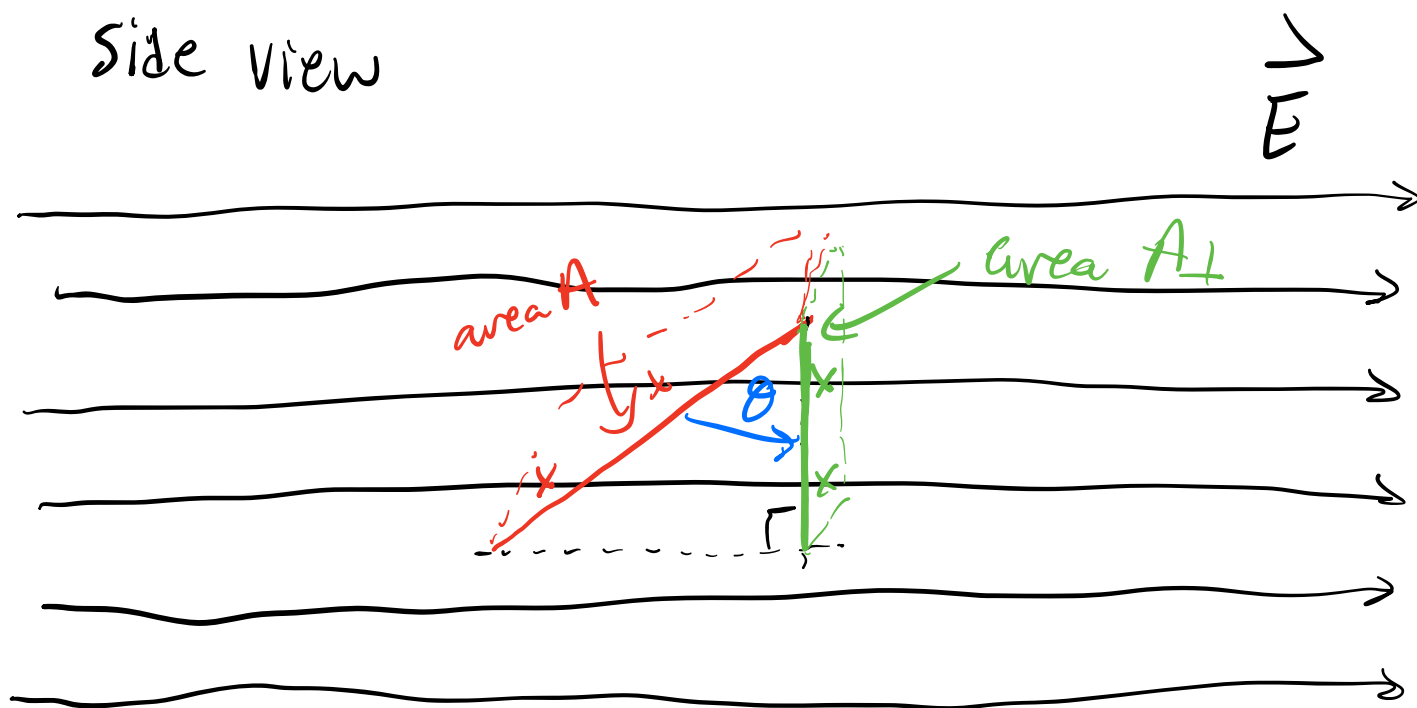


In this simple case, the electric flux is given by

$$\boxed{\Phi = EA}$$

Next, imagine the same \vec{E} & same surface, except we now tilt the surface relative to dir'n of \vec{E} .

Side view



When surface is not \perp to \vec{E} , flux is reduced b/c fewer lines passing through surface.

We will define an "equivalent" surface that has the same flux through it.

Green & red area are related by $\cos \theta$

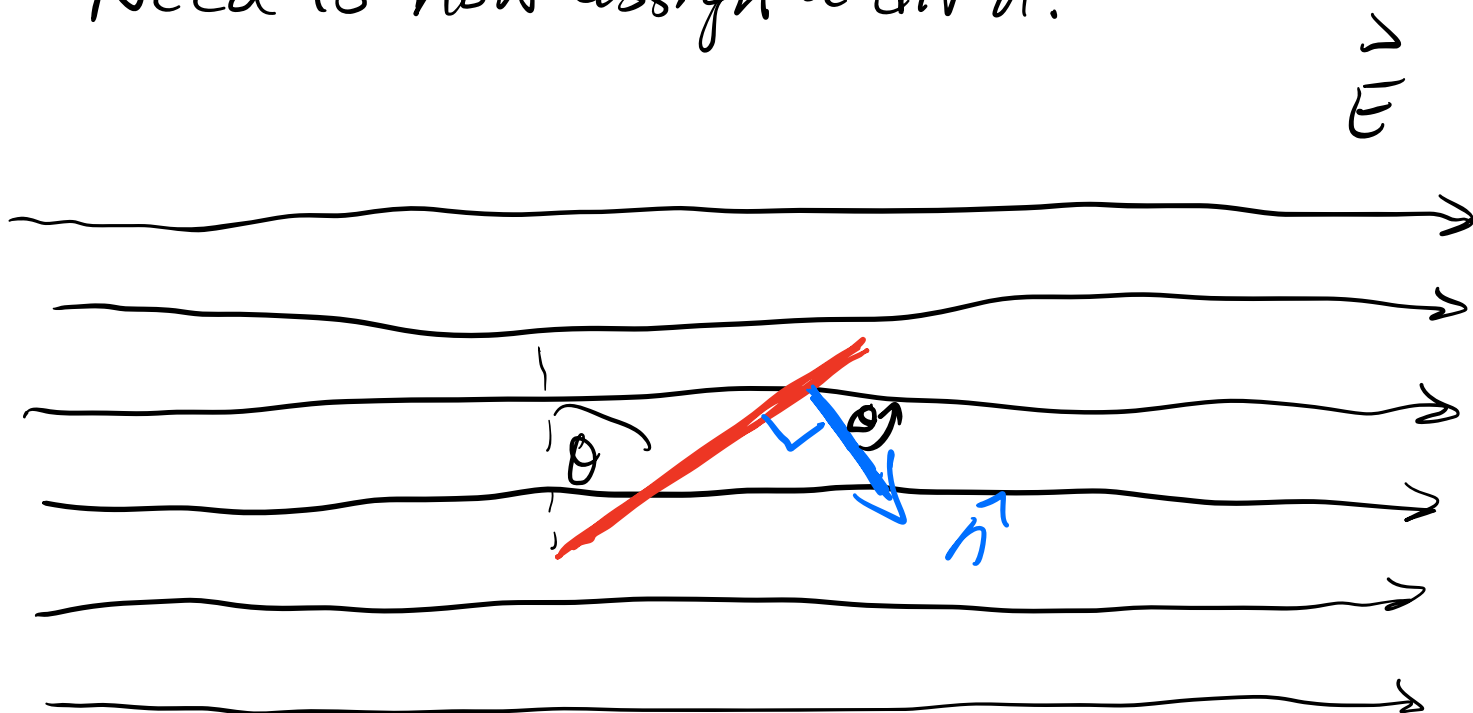
$$A_{\perp} = A \cos \theta$$

Since A_{\perp} is \perp to uniform \vec{E} ,
know $\boxed{\Phi = EA_{\perp} = EA \cos \theta}$

We would like to express Φ using vector quantities. \vec{E} is already a vector, so need to define an area vector \vec{A} .

Choose $|\vec{A}| = A$ (area).

Need to now assign a dir'n.



Define \hat{n} to be a unit vector (length 1) that is \perp to our surface.

Consider $\vec{E} \cdot \vec{A} = \vec{E} \cdot (A \hat{n})$

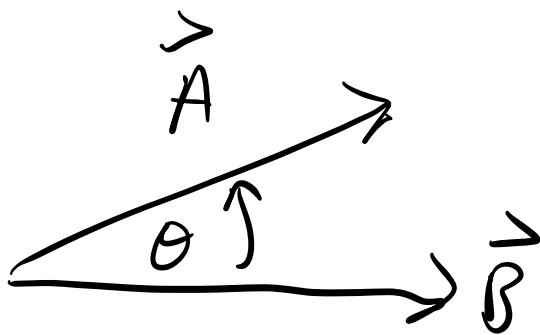
$$= A \vec{E} \cdot \hat{n}$$

$$= A \underbrace{|\vec{E}|}_{E} \underbrace{|\hat{n}|}_{1} \cos \theta$$

$$E \cdot 1$$

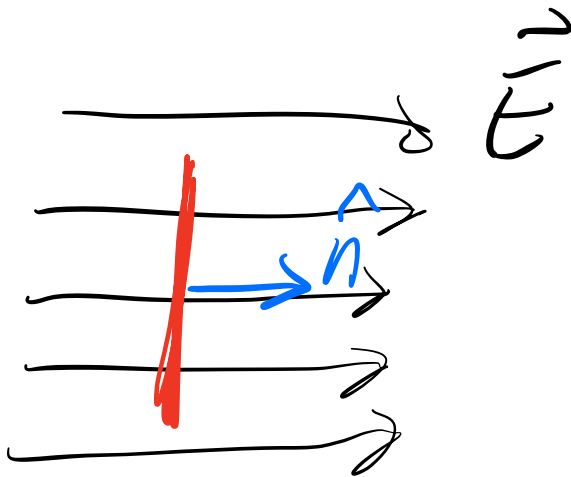
$$\Phi = \vec{E} \cdot \vec{A} = EA \cos \theta$$

dot product $\vec{A} \cdot \vec{B} = AB \cos \theta$



Test $\Phi = \vec{E} \cdot \vec{A} = EA \cos \theta$

$\theta = 0$ case

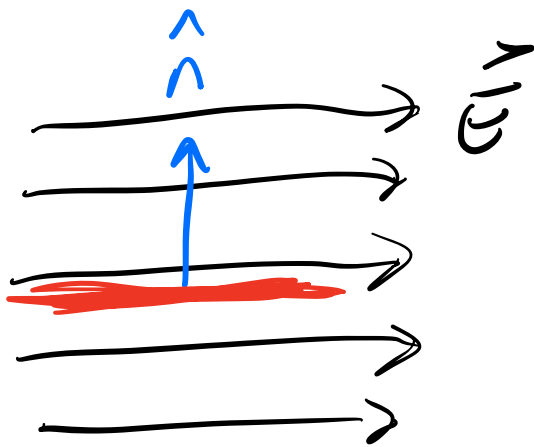


$$\Phi = EA \cos 0$$

1

$$\therefore \Phi = EA \quad \checkmark$$

$\theta = 90^\circ$ case

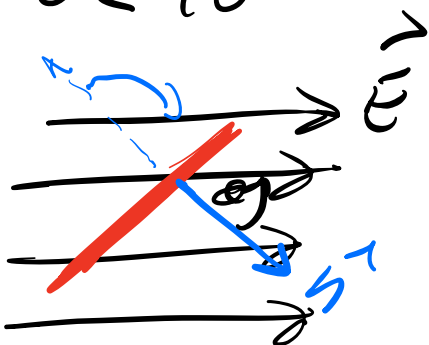


$$\Phi = \vec{E} \cdot \vec{A} = EA \cos 90^\circ$$

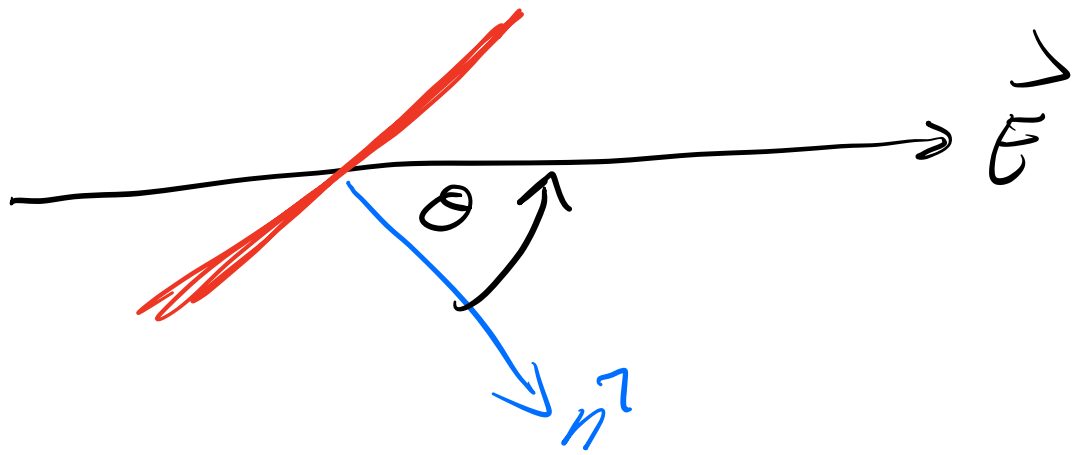
0

$$= 0 \quad \checkmark$$

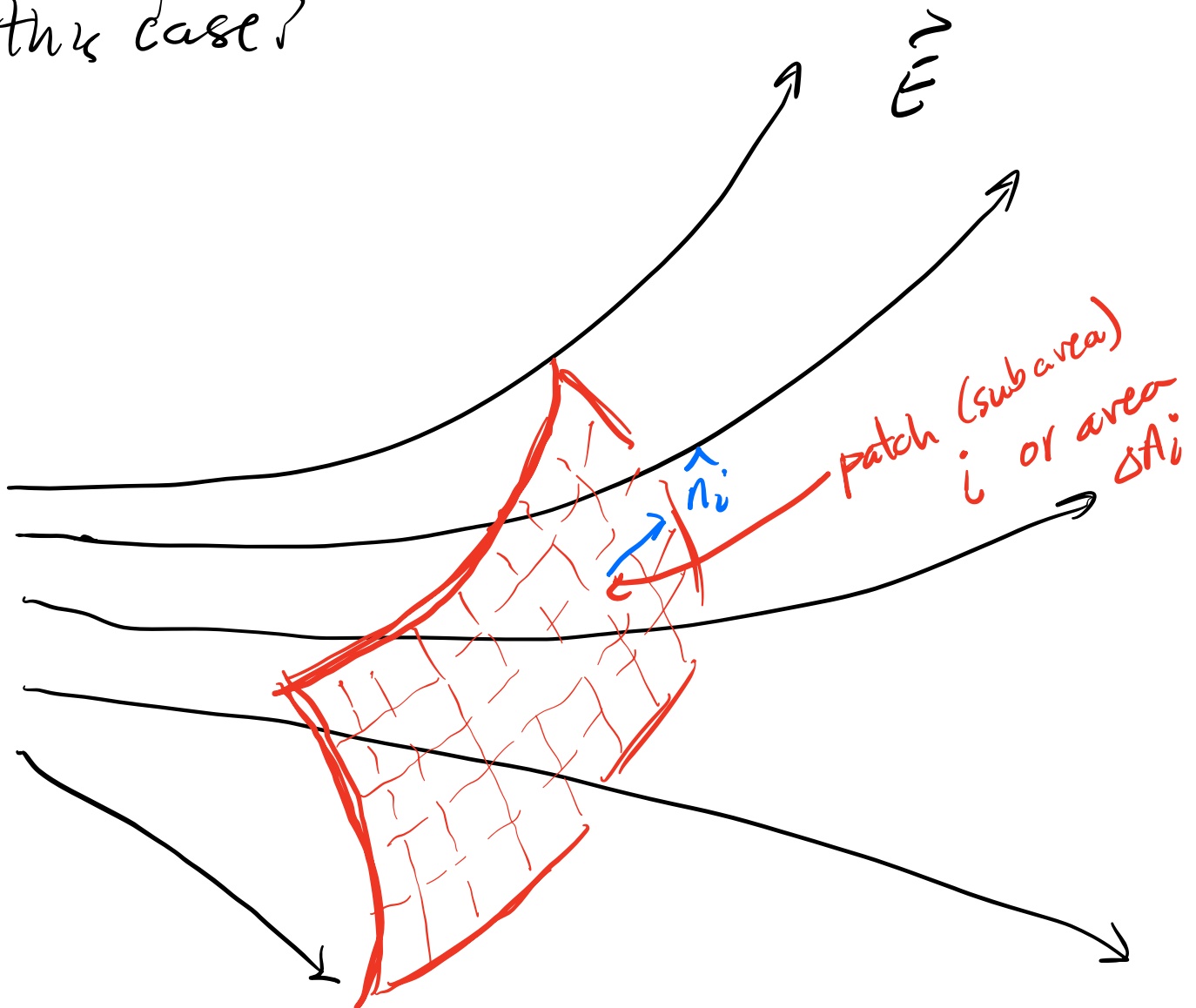
$0 < \theta < 90^\circ$



$$\Phi = EA \cos \theta$$



What if \vec{E} is not uniform & our surface is not flat? How do we calc. the flux in this case?



To handle this case, we divide large area into small subareas s.t. each subarea is approx. flat & \vec{E} over a subarea is approx. uniform.

Then for subarea i , the flux is

$$\begin{aligned}\Phi_i &= E_i \Delta A_i \cos \theta_i \\ &= \vec{E}_i \cdot \vec{\Delta A}_i\end{aligned}$$

$$\Phi = \sum_i \Phi_i = \sum_i \vec{E}_i \cdot \vec{\Delta A}_i$$

In the limit $\Delta A_i \rightarrow 0$

$$\Phi = \int \vec{E} \cdot d\vec{A}$$