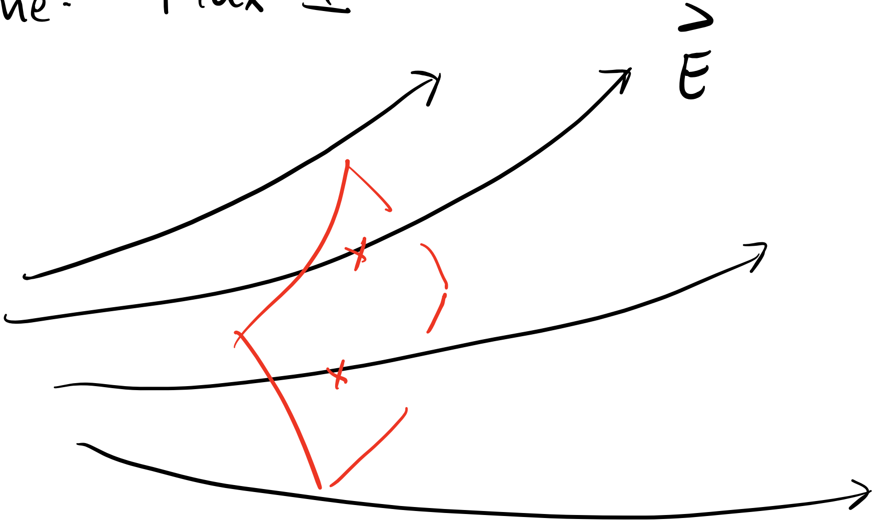
PHYS 121 Jan. 26, 2024

<u>To do</u>: - Complete HW3 by 23:59 today

- Complete Pre-Lab #1 before Lab #1 next week.
- Quiz #1 on PrairieLearn Wednesday, January 31.

Last Time: Flux Φ



Φ is proportional to number of $\vec{E}$-field lines passing through a surface.

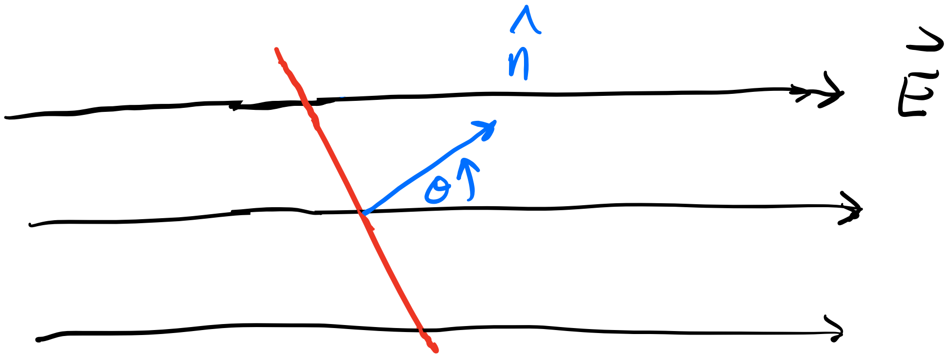
▨ Flat Surface ⊥ to uniform electric field

$$\Phi = EA$$

▨ Flat Surface at arbitrary angle in a uniform electric field

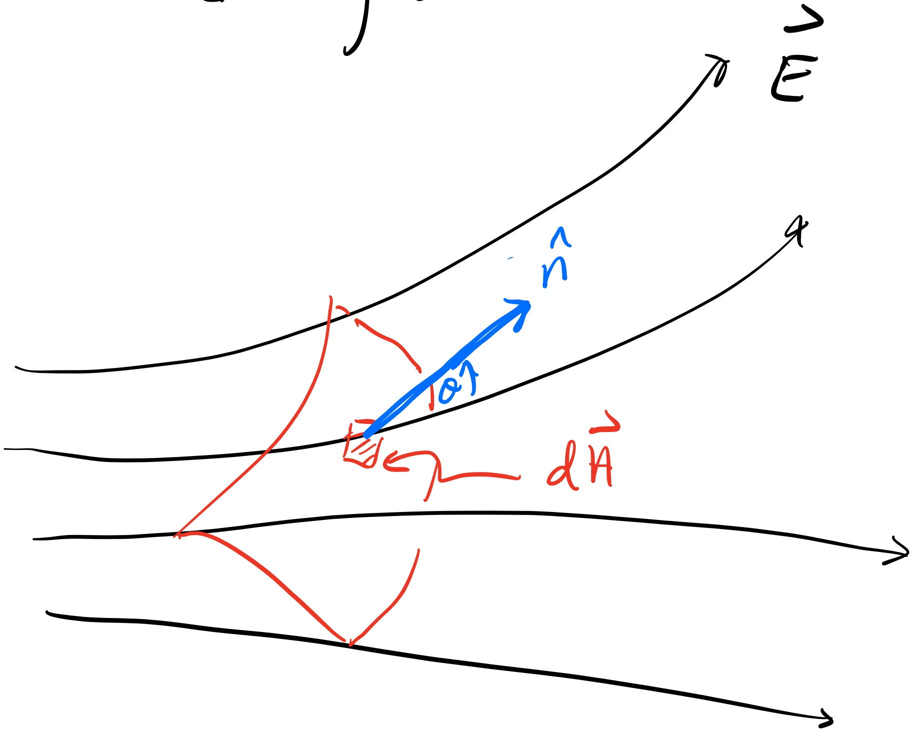
$$\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta$$

$$\vec{A} = A\hat{n}$$

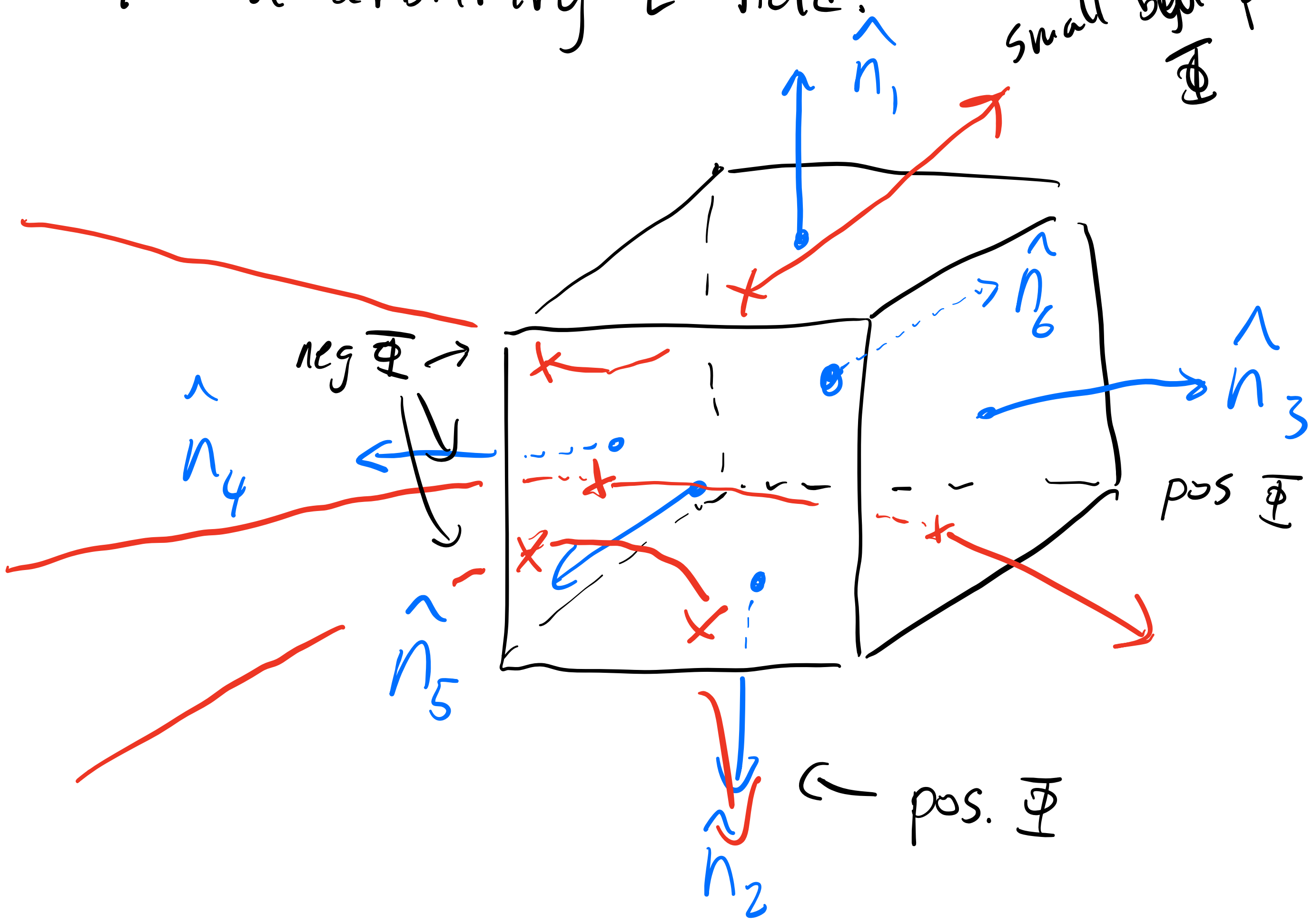


▨ Curved surface in a non-uniform magnetic field

$$\Phi = \int \vec{E}\cdot d\vec{A}$$



Let's now consider a closed surface in an arbitrary $\vec{E}$-field.



For closed surfaces, define our normal unit vectors s.t. they point outwards

On the left, $\hat{n}_4$ points left while $\vec{E}$ points right. $\therefore$ angle between $\hat{n}_4$ & $\vec{E}$ is approx 180°

$$\vec{E} \cdot \vec{A}_4 = \vec{E} \cdot (A_4 \, \hat{n}_4)$$

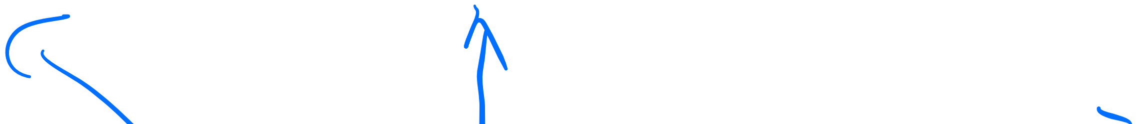
$$= EA_4 \underbrace{\cos(\simeq 180^\circ)}_{\simeq -1}$$

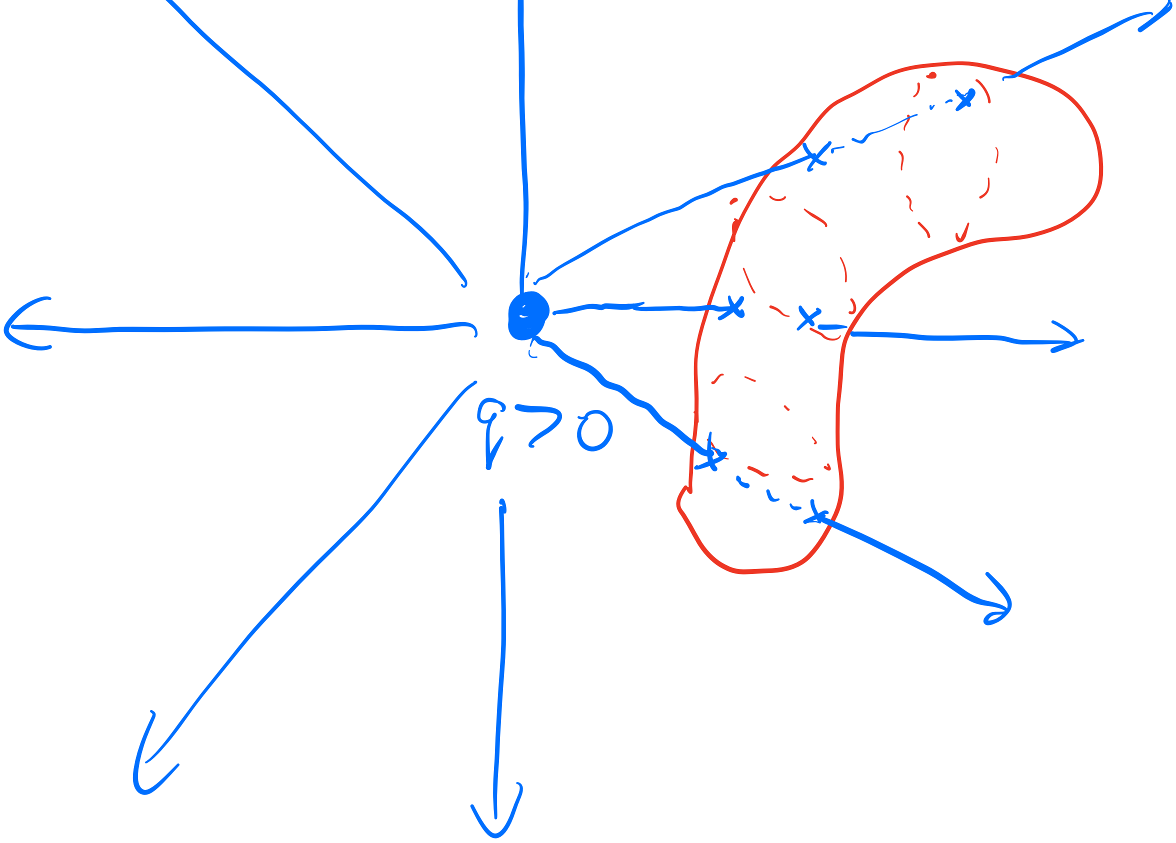
When $\vec{E}$ & $\hat{n}$ are anti-parallel, they contribute negative flux.

On the other hand, when $\vec{E}$ & $\hat{n}$ are approx parallel ($0 < \theta < 90$), get pos. contributions to Φ.

Notice that the cube surface has 3 field lines entering it (negative flux) & 3 field lines exiting (pos. flux). The net flux through this closed surface is zero.

Think about a pt. charge next to a closed surface of any shape.

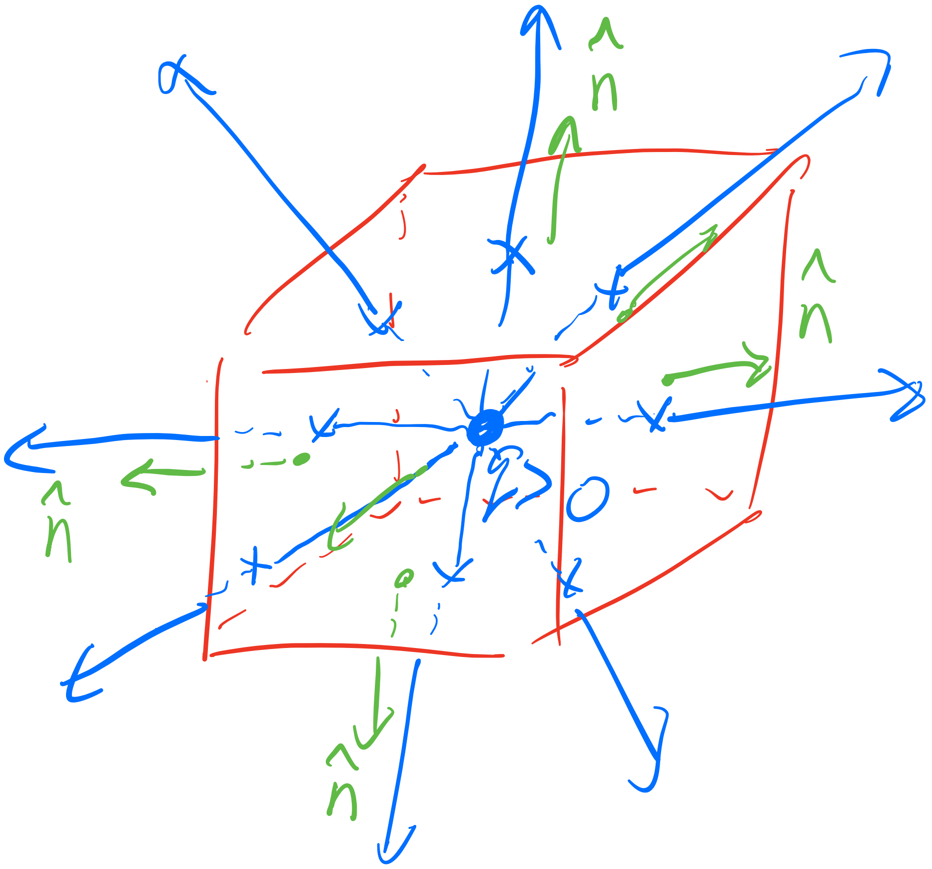




Since the same number of $\vec{E}$-field lines enter & exit the surface, the net flux is zero.

Any charge dist'n outside a closed surface of any shape contributes zero net flux b/c all field lines enter & exit the surface.

Think about a charge inside a closed surface.

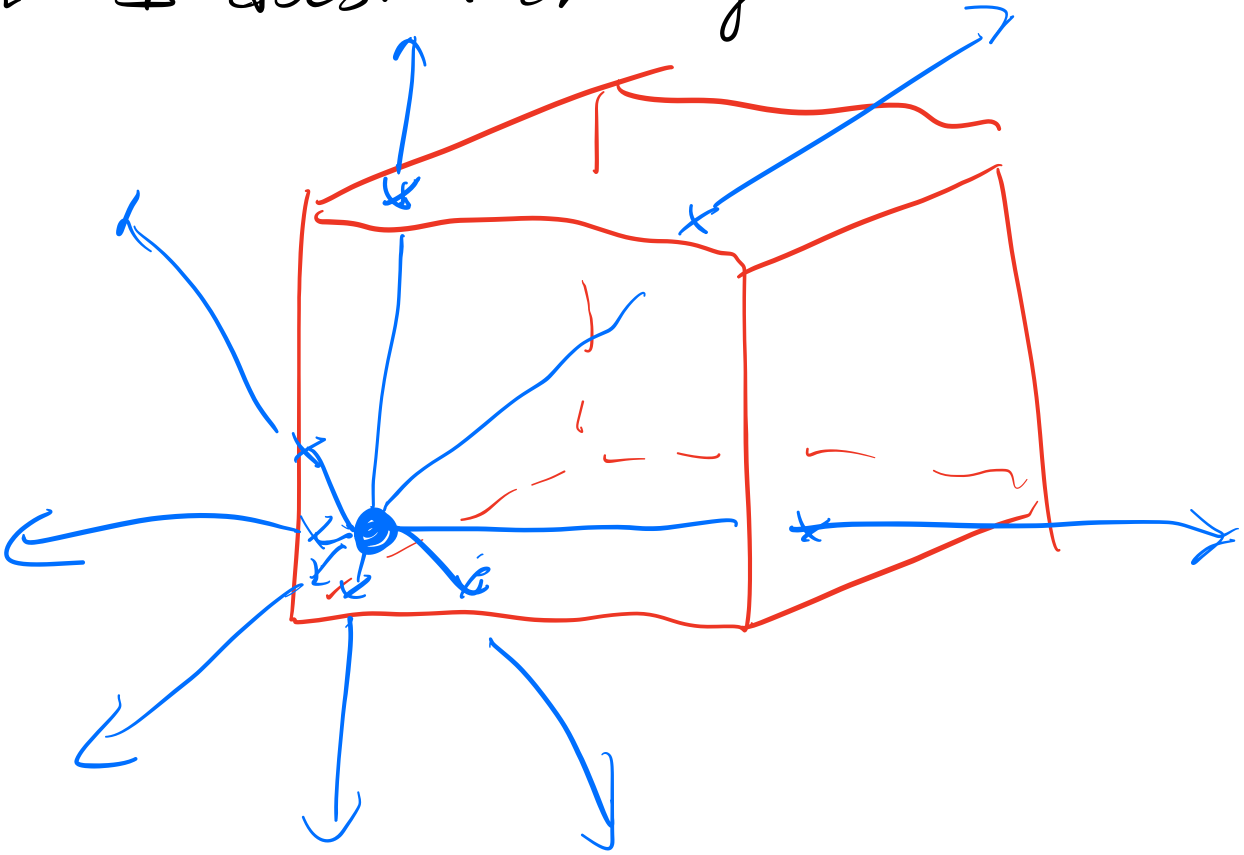


In this case, the field lines only exit the closed surface & we only get positive flux.

$$\Phi > 0.$$

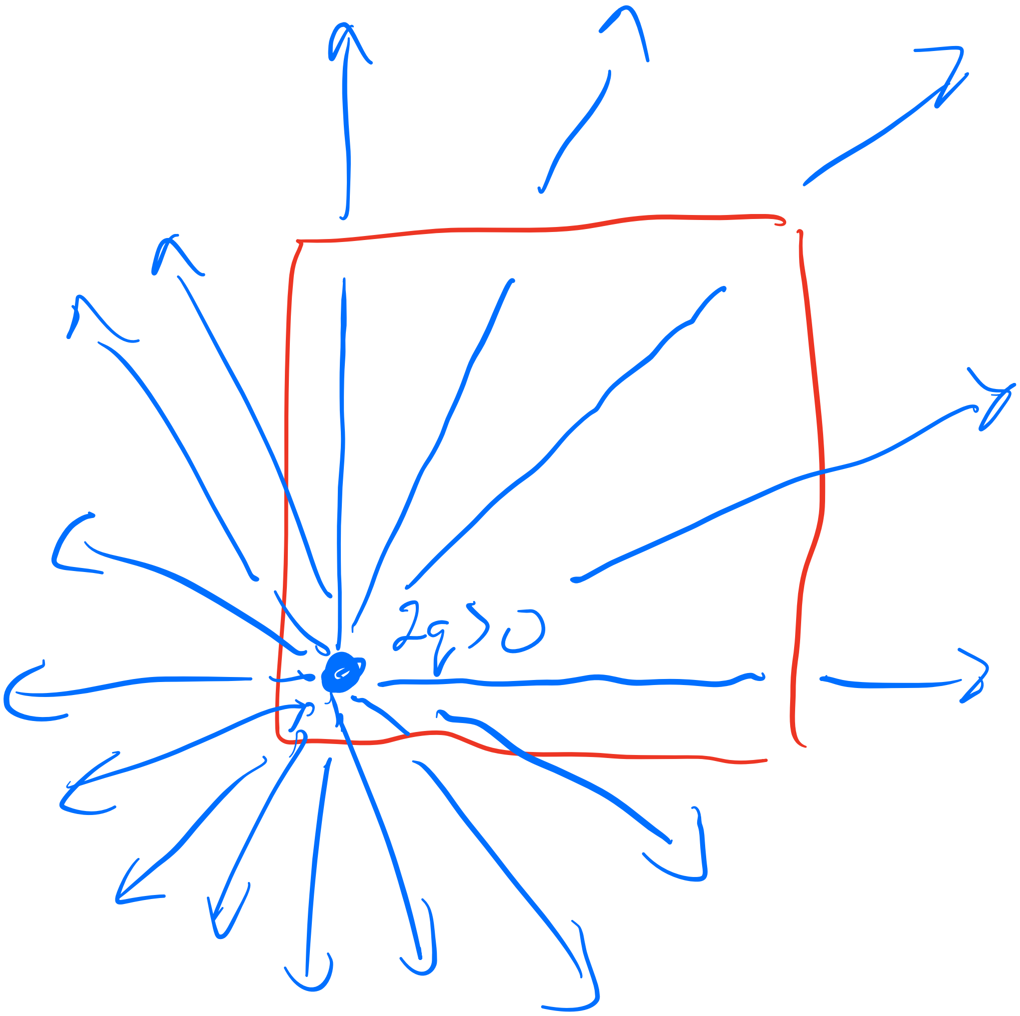
If we put $q < 0$ (negative) in a closed surface, get a non-zero flux $\Phi < 0$.

If we move the charge off centre of the cube, still have the same no. of $\vec{E}$-field lines exiting the closed surface & ∴ Φ doesn't change.



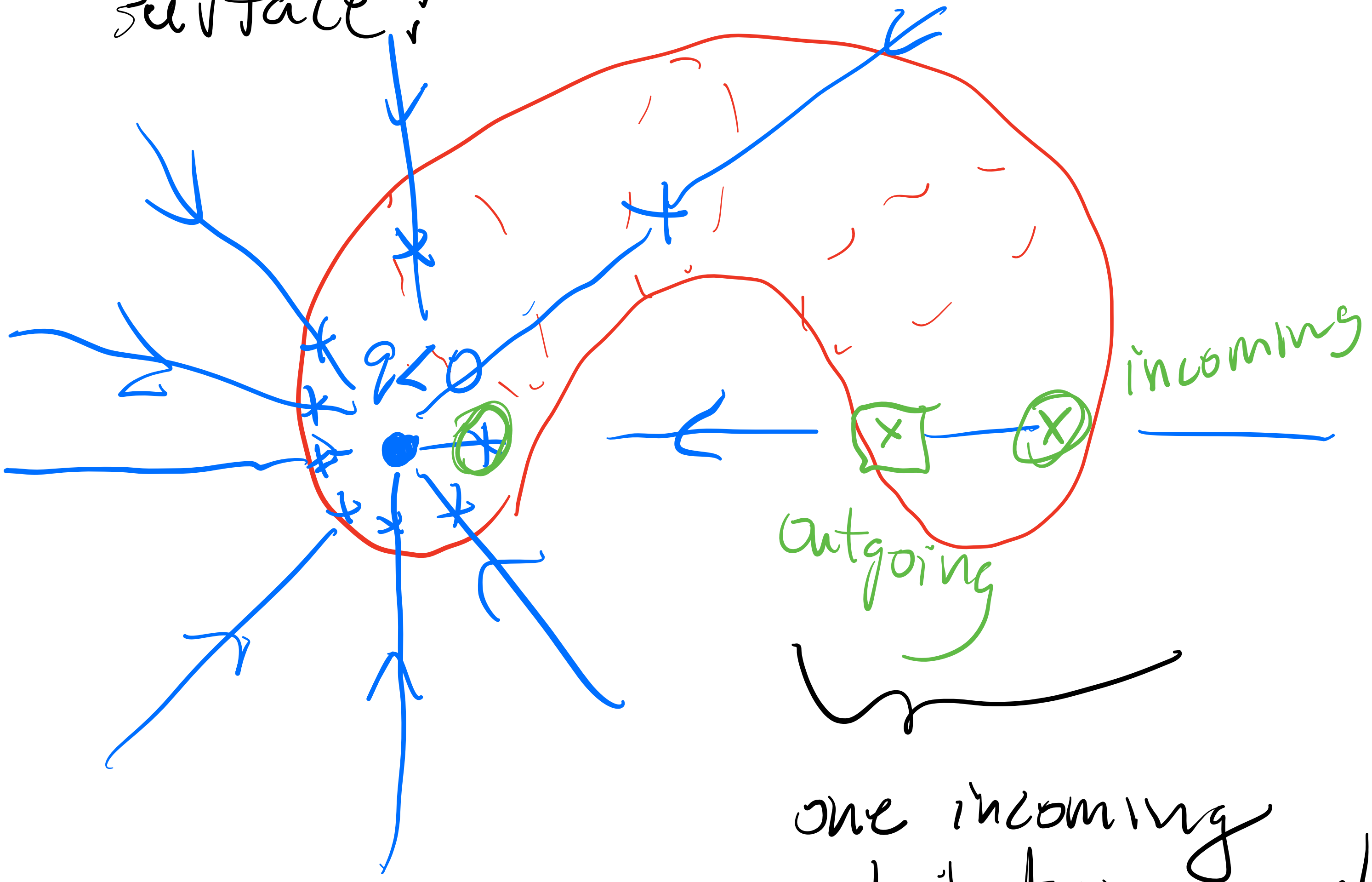
If we double the value of q in our closed surface, the $\vec{E}$ field doubles, get twice as many field lines.

Side view



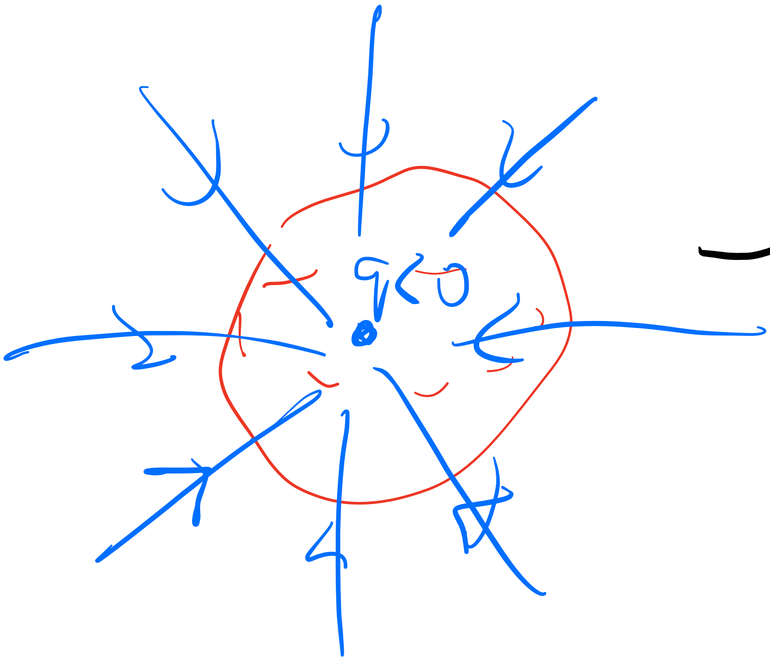
In this case Φ doubles b/c twice as many field lines exiting our surface.

What if we change shape of closed surface?



one incoming contribution cancels with one outgoing contribution

same flux Φ in these two cases.



⇒ Φ through a closed surface is prop. to. the enclosed charge.

Next time, we will derive Gauss's law. The result will be that:

① Already know $\Phi = \int \vec{E} \cdot d\vec{A}$

② Will find $\Phi = \frac{q_{encl}}{\varepsilon_0}$ for closed surface

Gauss's Law ⇒

$$\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}}$$

for closed surfaces

closed surface.

q_{encl}: total charge enclosed by the surface.