PHYS 121 Jan. 29, 2024

To do: - Complete HW4 by 23:59 on Friday

- Complete Pre-Lab #1 before start of Lab #1

- Quiz #1 on PrairieLearn Wednesday, January 31. You do not have to come to EME 0050 to complete the quiz.

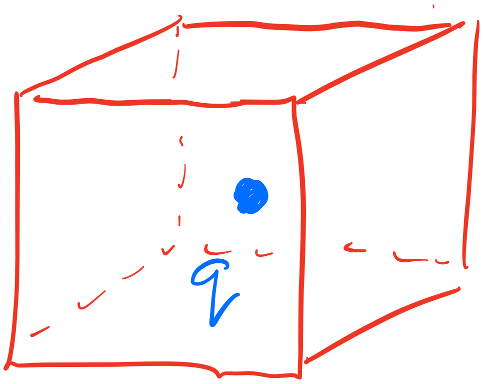
Last Time:

$$\Phi = \oint \vec{E} \cdot d\vec{A} \qquad (1)$$

closed surface

Φ is:
- ✓ ☒ prop. to total charge enclosed by surface
- ✓ ☒ indep. of position of charge enclosed by surface
- ✓ ☒ Indep. of surface shape.

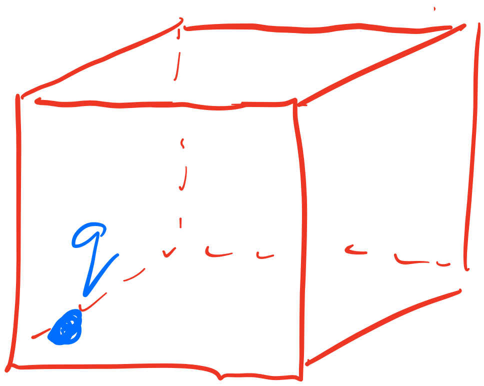
1.



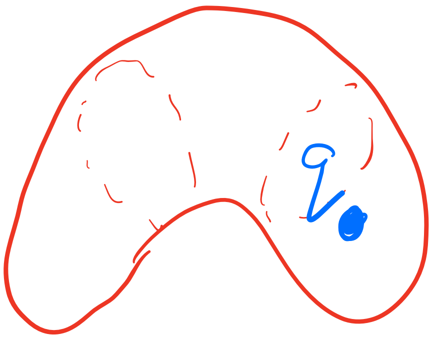
If q is the same in all three cases, then

$$\boxed{\Phi_1 = \Phi_2 = \Phi_3}$$

2.



3.



Today: <u>Gauss's Law</u>

will show that $\Phi = \dfrac{q_{encl}}{\varepsilon_0}$ ②

Require ① = ② $\Rightarrow$ $\boxed{\oint \vec{E} \cdot d\vec{A} = \dfrac{q_{encl}}{\varepsilon_0}}$

Find flux through a closed surface containing a point charge q.

- free to pick shape of surface & position of pt. charge

- Make choices that simplify the calc.

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

▪ If $\vec{E} \parallel d\vec{A}$, then $\vec{E} \cdot d\vec{A}$

$$= E\,dA \underbrace{\cos 0}_{1}$$

$$= E\,dA$$

perpendicular

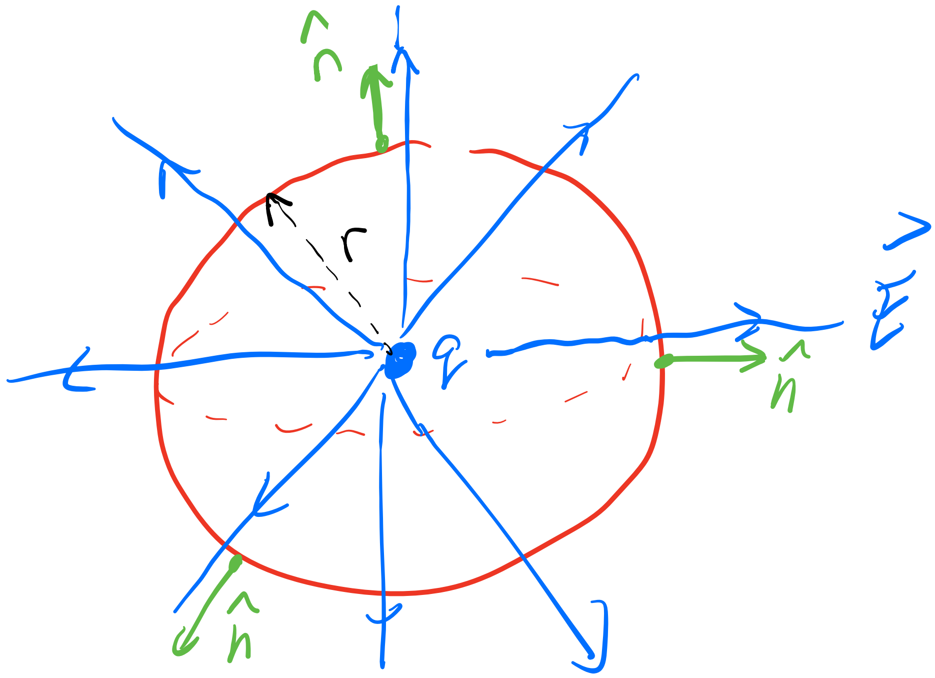
▪ If $\vec{E} \perp d\vec{A}$, then $\vec{E} \cdot d\vec{A}$

$$= E\,dA \underbrace{\cos 90^\circ}_{0}$$

$$= 0$$

▪ If $\vec{E}$ has const. mag. everywhere on surface, then we can take E outside integral.

- Choose a spherical surface w/ q at the centre of the sphere.



For this geometry $\vec{E} \parallel \hat{n}$

$$\therefore \vec{E} \cdot d\vec{A} = E\,dA \underbrace{\cos\theta}_{1}$$

$$= E\,dA.$$

Note that pt. charge electric field has mag. $\boxed{k_e \frac{q}{r^2}}$ at surface of sphere.

$\therefore$ E has a <u>const</u> mag. everywhere on surface of sphere $\rightarrow$ factor E out of integral.

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \underbrace{\oint E\, dA}_{\vec{E} \parallel d\vec{A}} = \underbrace{E \oint dA}_{E \text{ is const.}}$$

Note $\oint dA$ requires us to add up all the little patches of area that make entire surface of sphere.

$$\oint dA = \text{surface area of sphere}$$

$$= 4\pi r^2 .$$

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \underbrace{E}_{①} (4\pi r^2)$$

$$\frac{1}{\cancel{4\pi}\varepsilon_0}$$

$$\Phi = \frac{k_e q}{\cancel{r^2}} (4\pi \cancel{r^2}) = \cancel{4\pi} k_e q$$

$$\boxed{\Phi = \frac{q}{\varepsilon_0}}$$

Valid for any position of q inside a closed surface of any shape.

To emphasize that q must be inside closed surface, usually add a subscript q_{encl}.

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$$\Phi = \frac{q_{encl}}{\varepsilon_0}$$

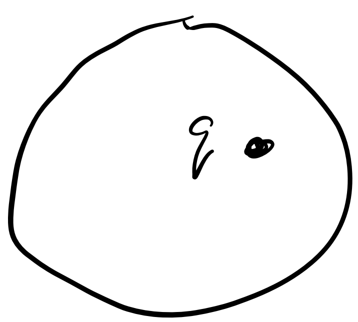
$$\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}}$$

Gauss's Law.

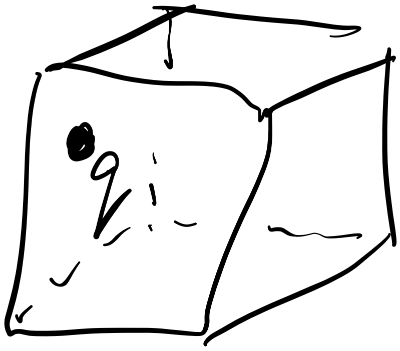
Since we can form any arbitrary dist'n of charges by assembling a collection of pt charges, Gauss's is valid for any kind of charge dist'n (not just pt. charges).

Eg. What is Φ?

$$\Phi = \frac{q}{\varepsilon_0}$$

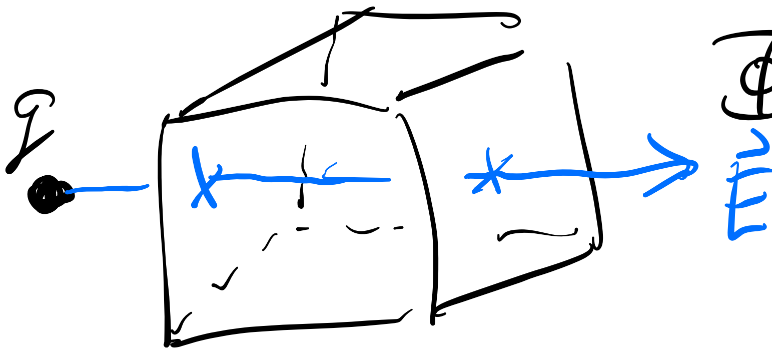


Eg.



$$\Phi = \frac{q}{\varepsilon_0}$$

Eg



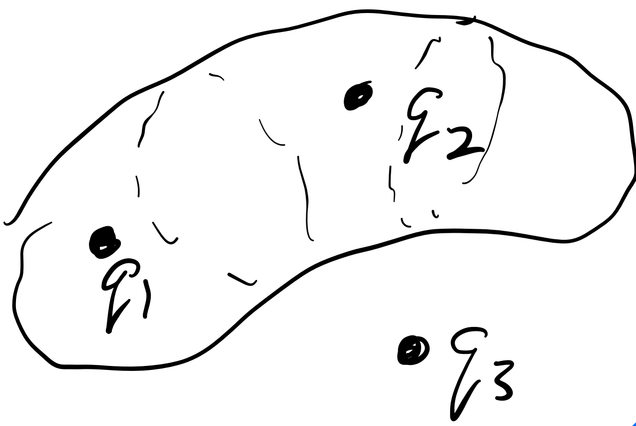
$\Phi = 0$

since

$q_{encl} = 0$

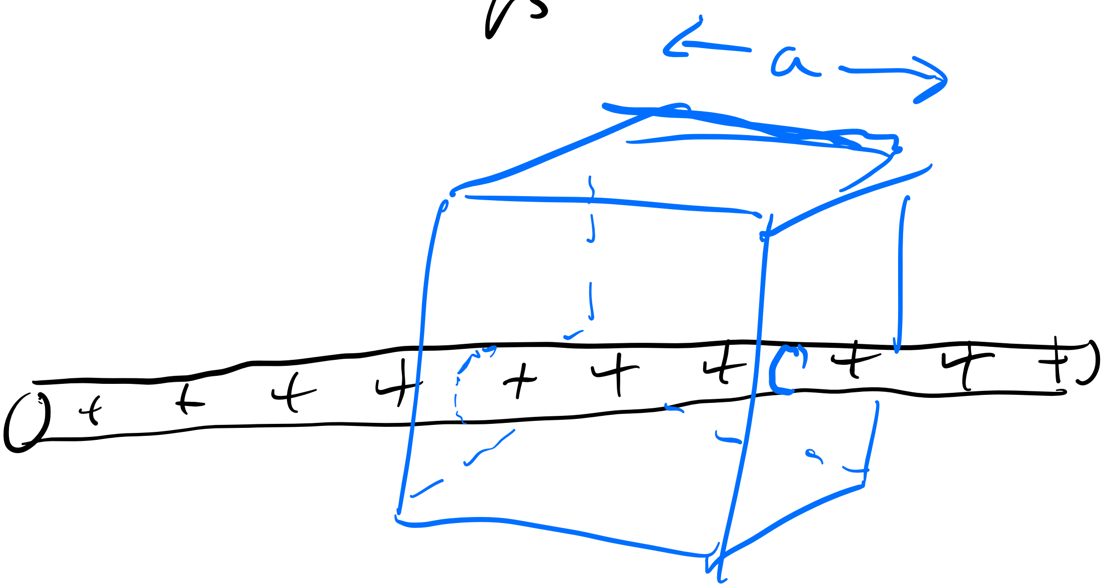
(q outside closed surface)

Eg.



$$\Phi = \frac{q_1 + q_2}{\varepsilon_0}$$

Eg.



charge per unit length of rod is $\lambda = \frac{Q}{L}$

What is Φ? $\Phi = \frac{q_{encl}}{\varepsilon_0}$

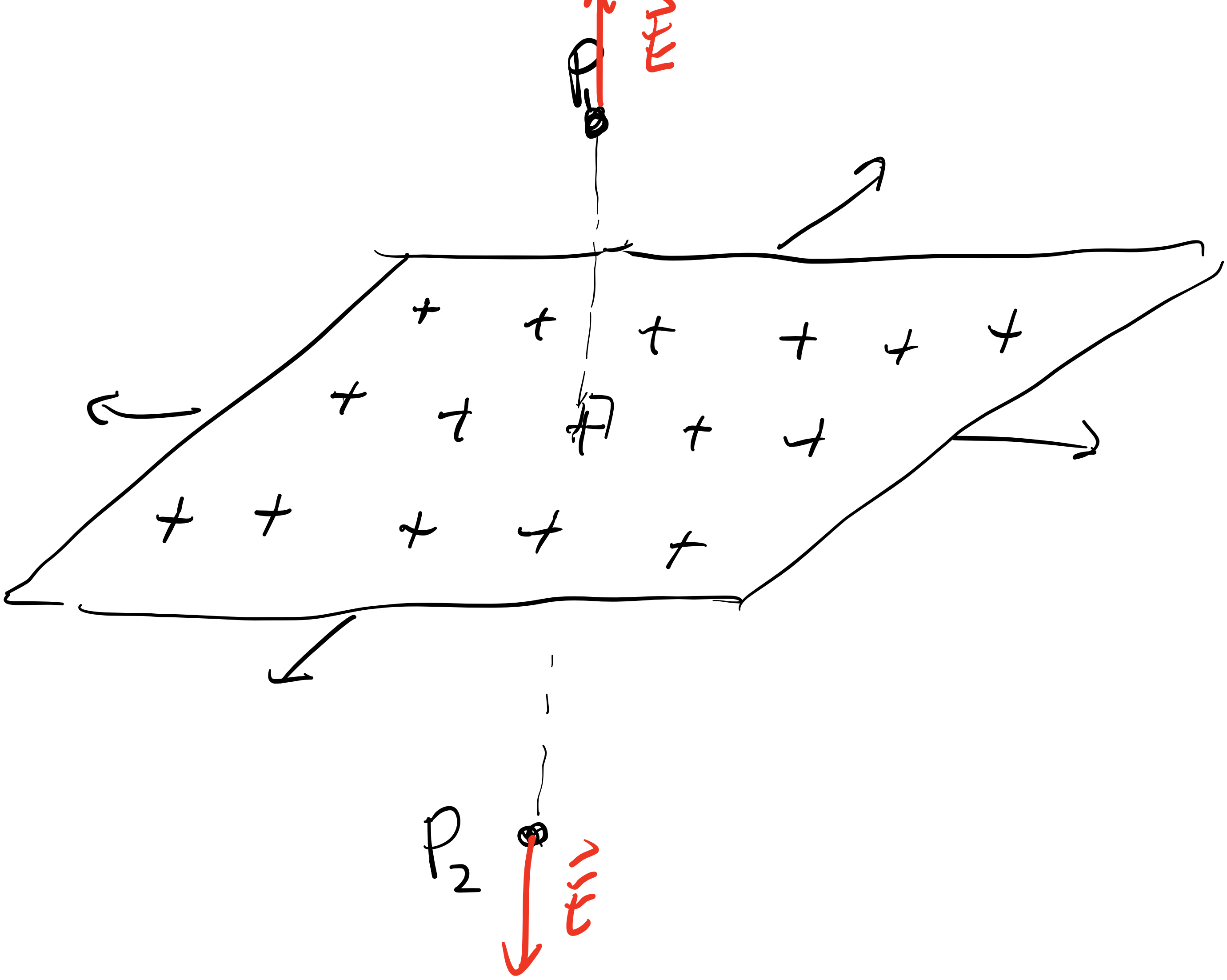
q_{encl} is total charge inside cube.

Need to mult. the charge per unit length of rod by the length of rod that is inside cube. That length is a.

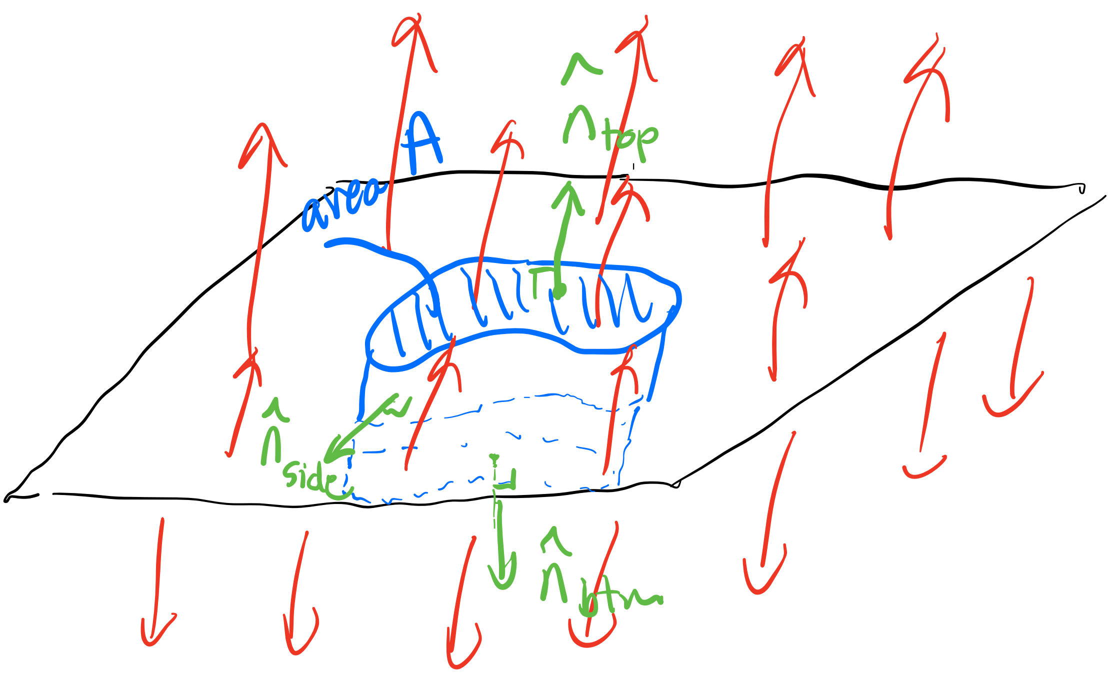
$q_{encl} = \lambda a$

$$\boxed{\Phi = \frac{\lambda a}{\varepsilon_0}}$$

Let's use Gauss's Law to find the electric field due to a uniformly charged sheet.



By symmetry $\vec{E}$ anywhere above sheet is upwards & it is downwards below the sheet.



Pick our surface s.t. it has flat top & btm that are always the same dist. from sheet.

$$\hat{n}_{top} \parallel \vec{E} \qquad \vec{E}\cdot d\vec{A}_{top} = E\,dA_{top}$$

$$\hat{n}_{btm} \parallel \vec{E} \qquad \vec{E}\cdot d\vec{A}_{btm} = E\,dA_{btm}$$

$$\hat{n}_{side} \perp \vec{E} \qquad \vec{E}\cdot d\vec{A}_{side} = 0$$

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \int \vec{E}\cdot d\vec{A}_{top} + \int \vec{E}\cdot d\vec{A}_{btm} + \cancel{\underbrace{\int \vec{E}\cdot d\vec{A}_{side}}_{0}}$$

$$= \int E\,dA_{top} + \int E\,dA_{btm}$$

Since top & btm surfaces are const dist. from sheet, E is const.

$$\Phi = E\underbrace{\int dA_{top}}_{A} + E\underbrace{\int dA_{btm}}_{A} = 2EA$$

$$\Phi = 2EA$$

$$\Phi = \frac{q_{encl}}{\varepsilon_0}$$

$$=$$

$$\Phi = \frac{\sigma A}{\varepsilon_0}$$

If sheet has charge per unit area σ.

Then $q_{encl} = \sigma A$

$$2E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

uniformly charged sheet.