PHYS 121 Feb. 2, 2024

<u>To do</u>: - Complete HW4 by 23:59 today

- No Pre-Lab #2.

Last Time: Gauss's Law

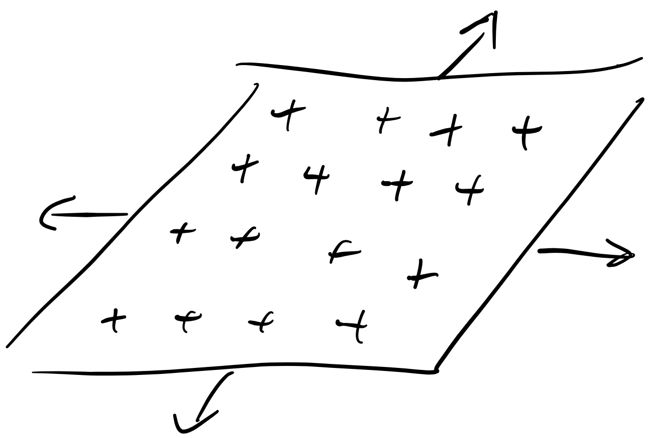
$$\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}}$$

Choose Gaussian surface s.t.

① $\vec{E} \parallel d\vec{A}$ or $\vec{E} \perp d\vec{A}$

② $|\vec{E}|$ is const over parts of surface where $\vec{E} \parallel d\vec{A}$

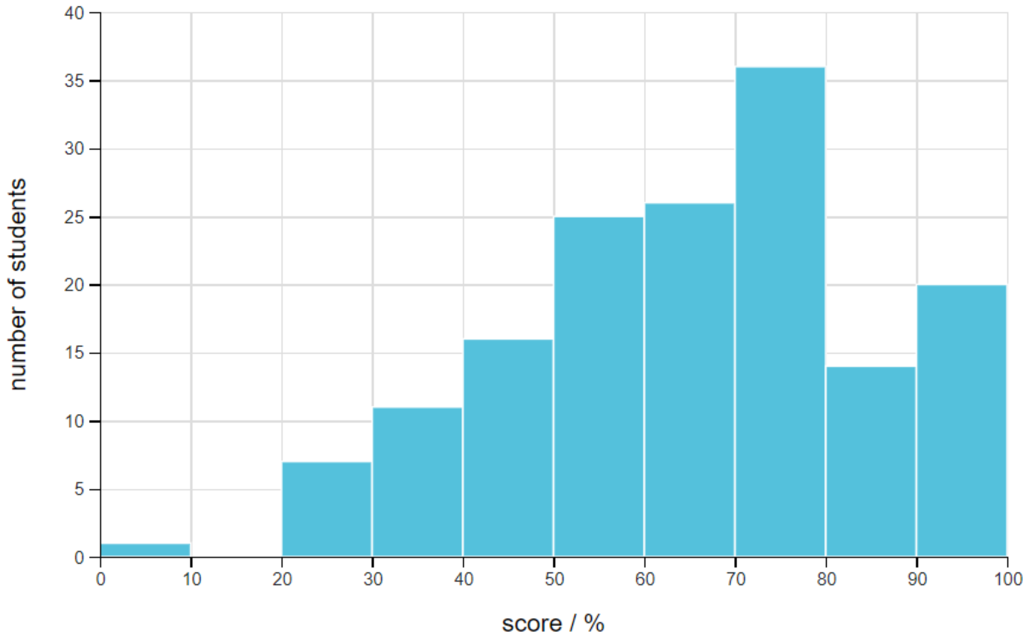
Sheet of charge



charge per unit area $\sigma = \dfrac{Q}{\text{area}}$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

indep. of position/distance from sheet.



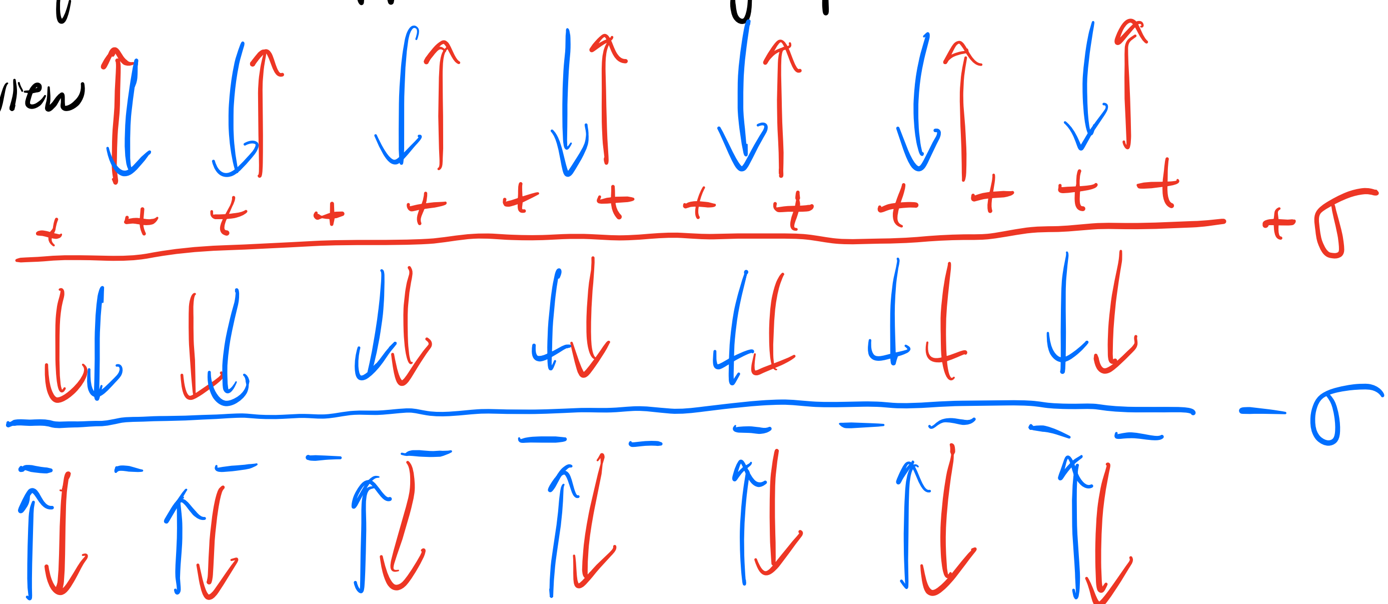
22% of class got 80% or better

Number of students	156
Mean score	64%
Standard deviation	19%

Sheet of uniform charge – Application
⇒ Parallel Plate Capacitor.

Consider two parallel sheets of charge w/ equal but opposite charge per unit area σ.

side view



We will first draw $\vec{E}$ due to pos. sheet.
Then draw $\vec{E}$ due to neg. sheet.
Net $\vec{E}$-field is the sum of the two.

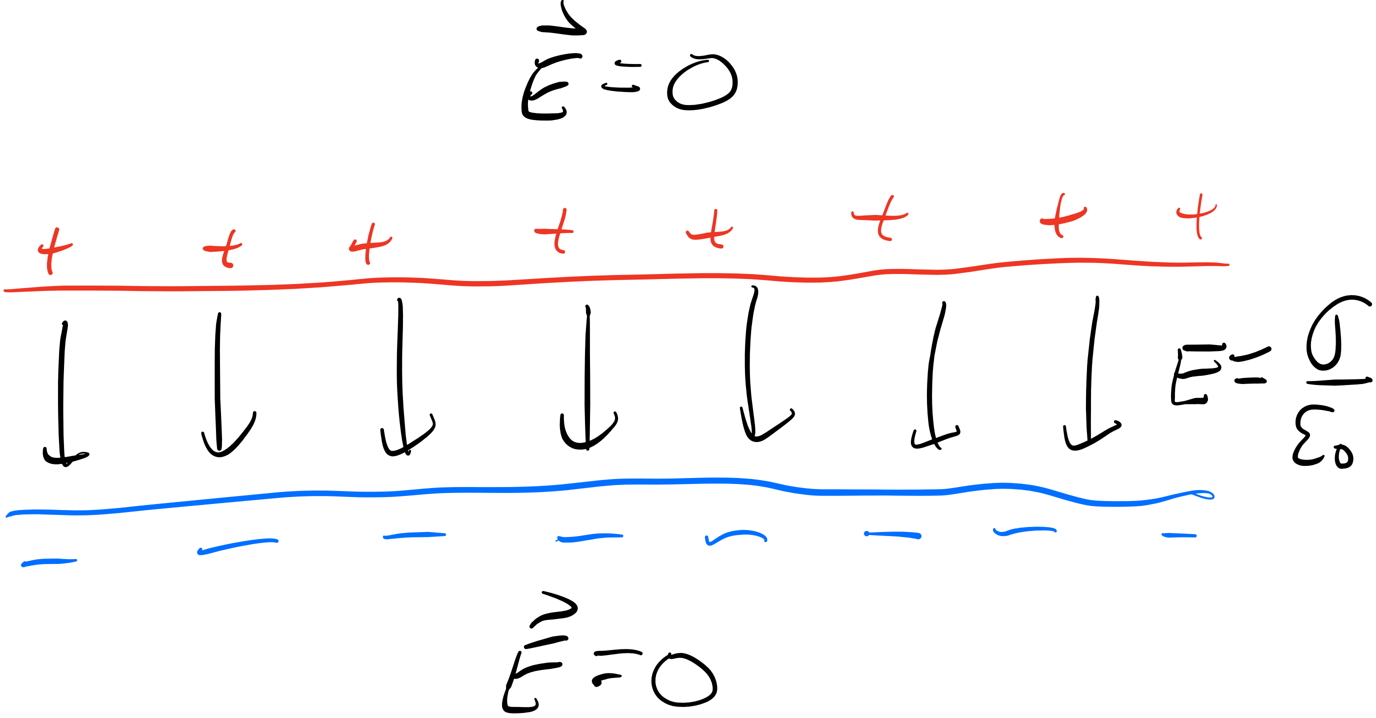
$$E_{sheet} = \frac{\sigma}{2\varepsilon_0}$$

Above & below the parallel sheets $\vec{E}_{net} = 0$ b/c the two contributions are equal in mag. & opposite in dir'n.

On the hand, the two contributions to $\vec{E}_{net}$ add between the sheets s.t.

$$\boxed{E_{net} = \frac{\sigma}{\varepsilon_0} \text{ between sheets.}}$$

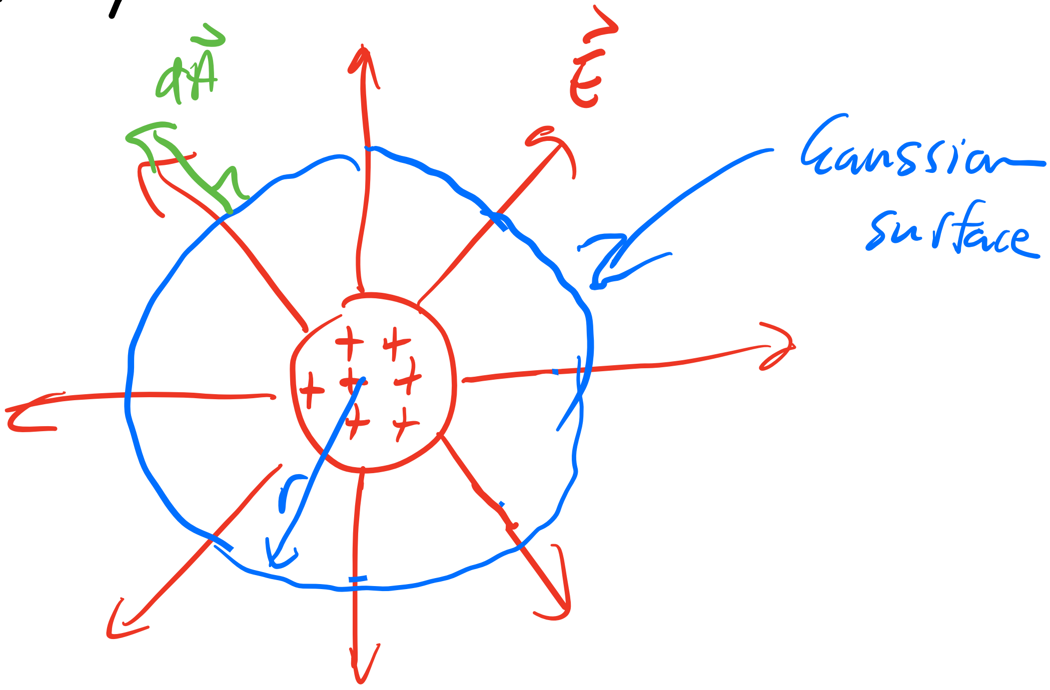
The parallel sheets of the capacitor gives us a way to create a uniform $\vec{E}$ between the plates w/ $\vec{E} = 0$ everywhere else.



Gauss's law is easy to apply in 3 scenarios

① Pt. charge or a spherical dist'n of charge.

Here, $\vec{E}$ will be in radial dir'n.



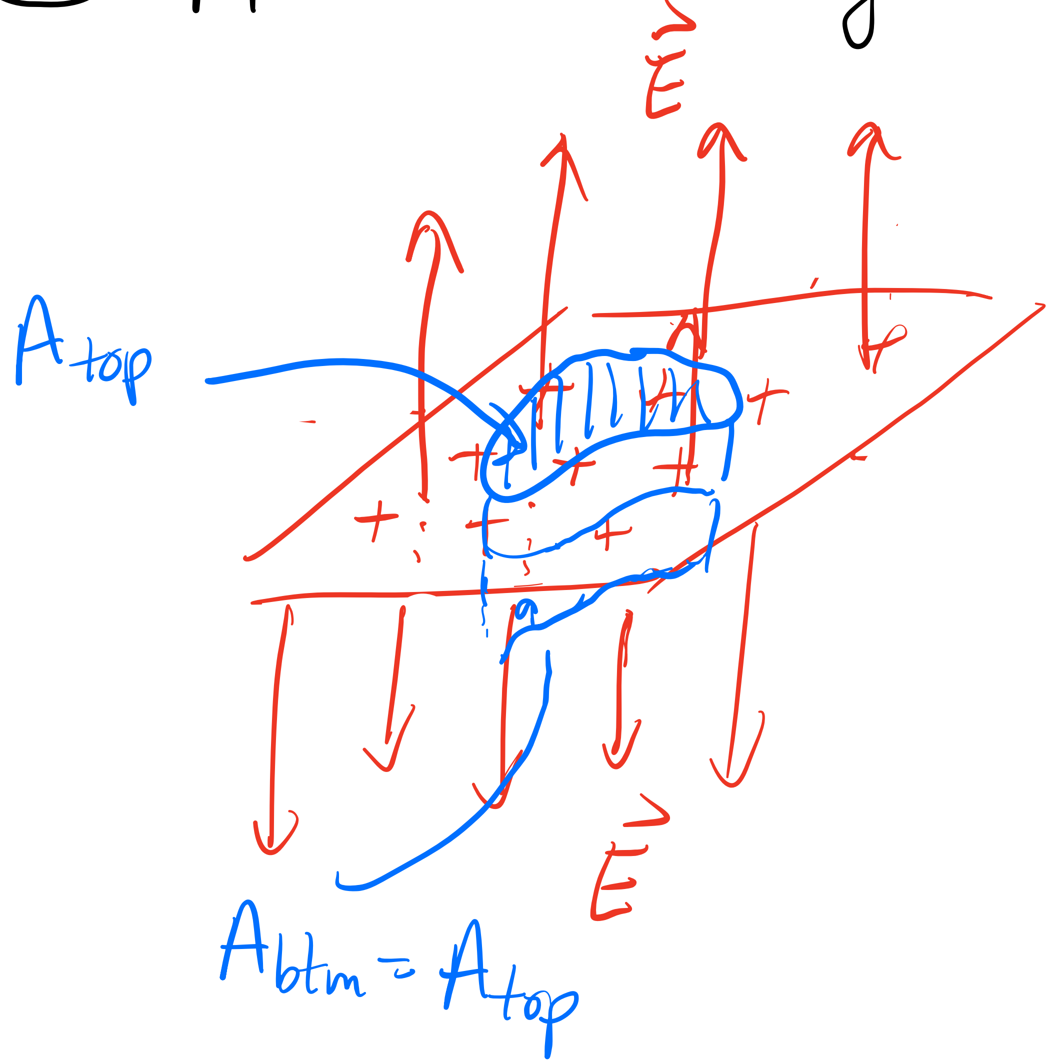
Select a spherical Gaussian surface s.t. $\vec{E}$ is const & $\perp$ at the surface.

$$\oint \vec{E}\cdot d\vec{A} = \oint E\,dA = E\oint dA$$

$$= E A_{sphere}$$

$$= E\,4\pi r^2$$

② Flat distn of charge



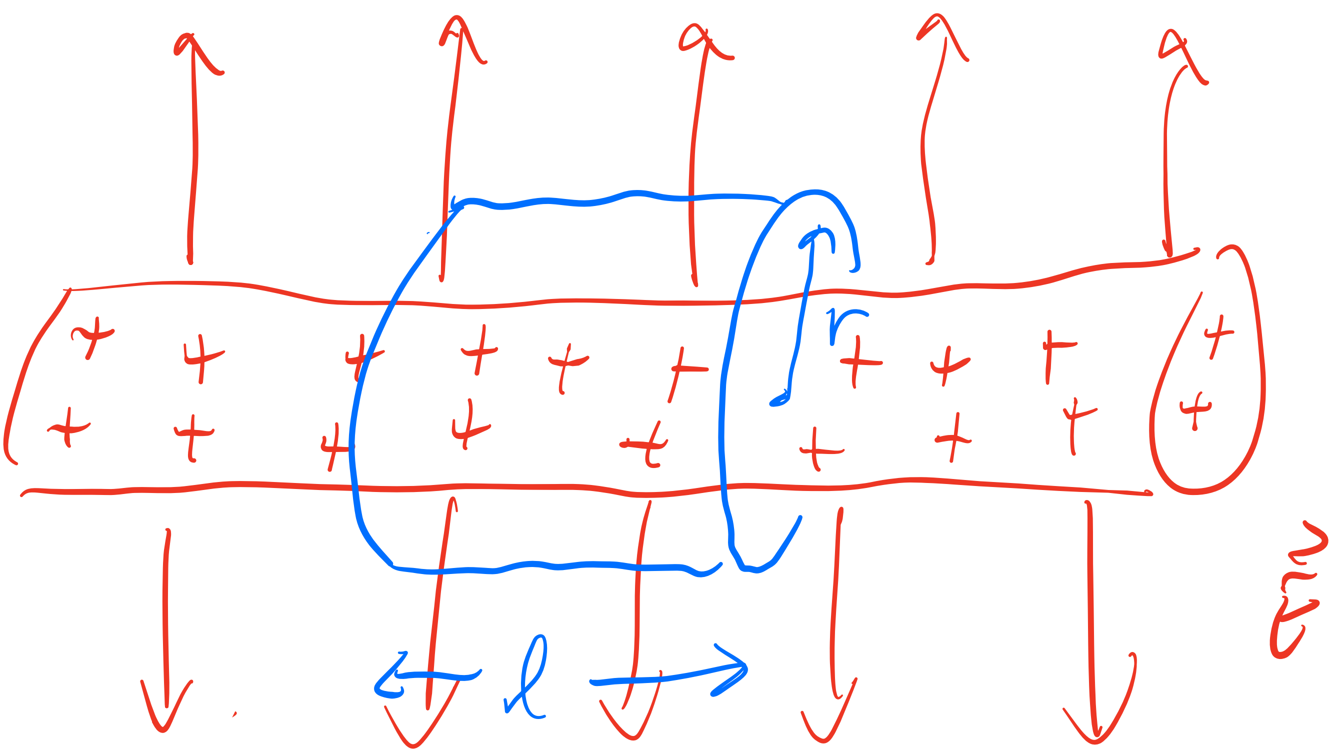
- $\vec{E} \perp$ to sheet.
- Select a Gaussian surface w/ flat top & btm

$$\oint \vec{E} \cdot d\vec{A} = \int_{top} \vec{E} \cdot d\vec{A} + \int_{btm} \vec{E} \cdot d\vec{A}$$

$$= 2 \int_{top} \vec{E} \cdot d\vec{A} = 2 \int_{top} E \, dA$$

$$= 2E \int_{top} dA = \boxed{2EA_{top}}$$

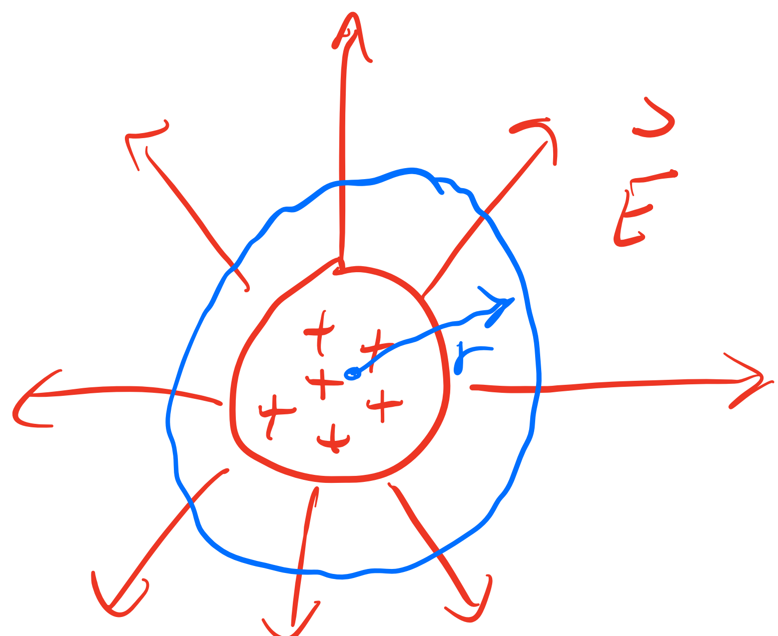
③ Cylindrical distn of charge. (v. long)



Assume rod is charge uniformly throughout its volume.

$$\rho = \frac{Q}{V}$$

Side View



Select a cylindrical gaussian surface.

Cylindrical surface has two side areas & a curved surface.

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \cancelto{0}{\int_{\substack{\text{left}\\ \text{side}}} \vec{E}\cdot d\vec{A}} + \cancelto{0}{\int_{\substack{\text{right}\\ \text{side}}} \vec{E}\cdot d\vec{A}}$$

$$+ \int_{\text{curved.}} \vec{E}\cdot d\vec{A}$$

$$\Phi = \int_{\text{curved}} \vec{E}\cdot d\vec{A} = \int_{\text{curved}} E\,dA = E\int_{\text{curved}} dA$$

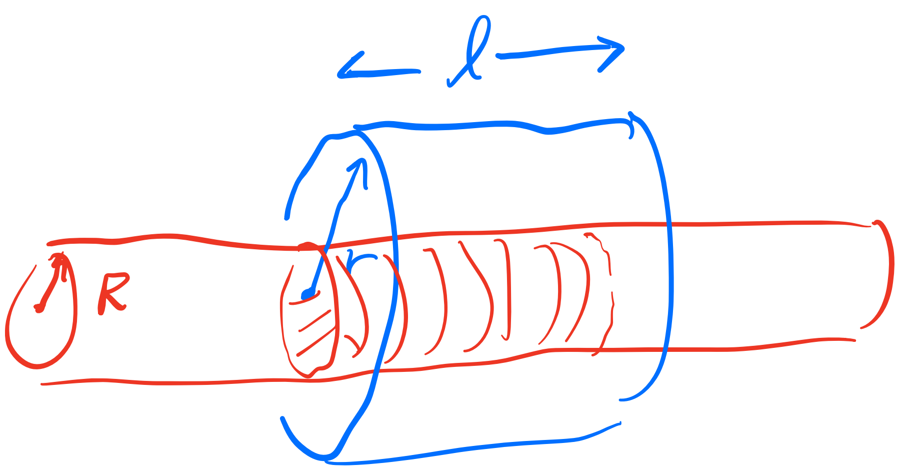
$$\Phi = E A_{curved} = \boxed{E(2\pi r \ell)}$$

Consider a long uniformly-charge rod of radius R & charge density ρ. Find $\vec{E}$

(a) a dist $r > R$ away from axis of rod (outside)

(b) a dist $r < R$ from axis of rod (inside).

(a)



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}$$

$E(2\pi r l)$

charge per unit volume of rod

$$q_{encl} = \rho \underbrace{\left(\text{volume of section of rod inside blue Gaussian surface}\right)}_{\pi R^2 l}$$

$q_{encl} = \rho \pi R^2 l$

Gauss's Law:

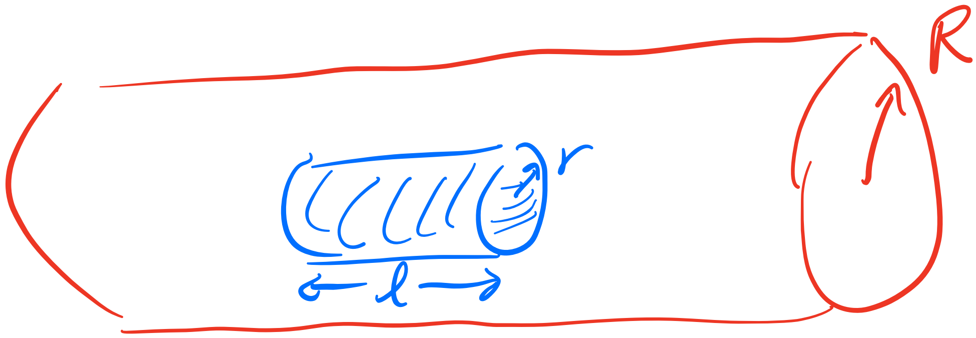
$$E(2\cancel{\pi} r \cancel{l}) = \frac{\rho \cancel{\pi} R^2 \cancel{l}}{\varepsilon_0}$$

$$\boxed{E = \frac{\rho R^2}{2\varepsilon_0 r}}$$

outside the rod

$r > R$

(b)



Just like in (a) $\oint \vec{E} \cdot d\vec{A} = E(2\pi r l)$

Now $q_{encl} = \rho \underbrace{\begin{pmatrix}\text{Blue shade} \\ \text{volume}\end{pmatrix}}_{\pi r^2 l}$

$$q_{encl} = \rho \pi r^2 l$$

$\therefore$ Gauss's law requires

$$E(\cancel{2\pi} \cancel{r} \cancel{l}) = \frac{\rho \cancel{\pi} r^{\cancel{2}} \cancel{l}}{\varepsilon_0}$$

$$\boxed{E = \frac{\rho r}{2\varepsilon_0}} \qquad r < R$$

points inside rod.