PHYS 121 Feb. 5, 2024

To do: - Complete HW5 by 23:59 on Friday

- No Pre-Lab #2

Last time: Integral in Gauss's Law

$$\oint \vec{E} \cdot d\vec{A}$$

① Spherical Symmetry

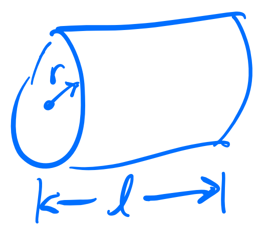
$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2)$$

② Planar Symmetry

$$\oint \vec{E} \cdot d\vec{A} = 2EA_{top/btm}$$

③ Cylindrical Symmetry

$$\oint \vec{E} \cdot d\vec{A} = E(2\pi r \ell)$$



To complete Gauss's Law, need to find q_{encl}/ε_0

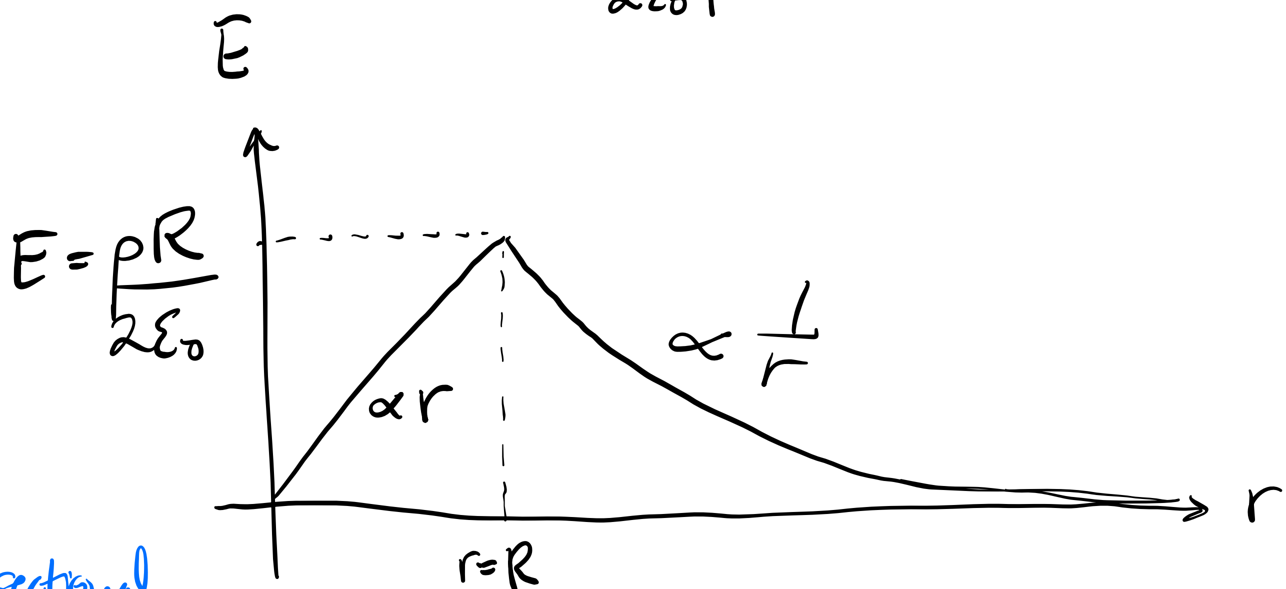
Uniformly-Charged Cylinder of radius R.

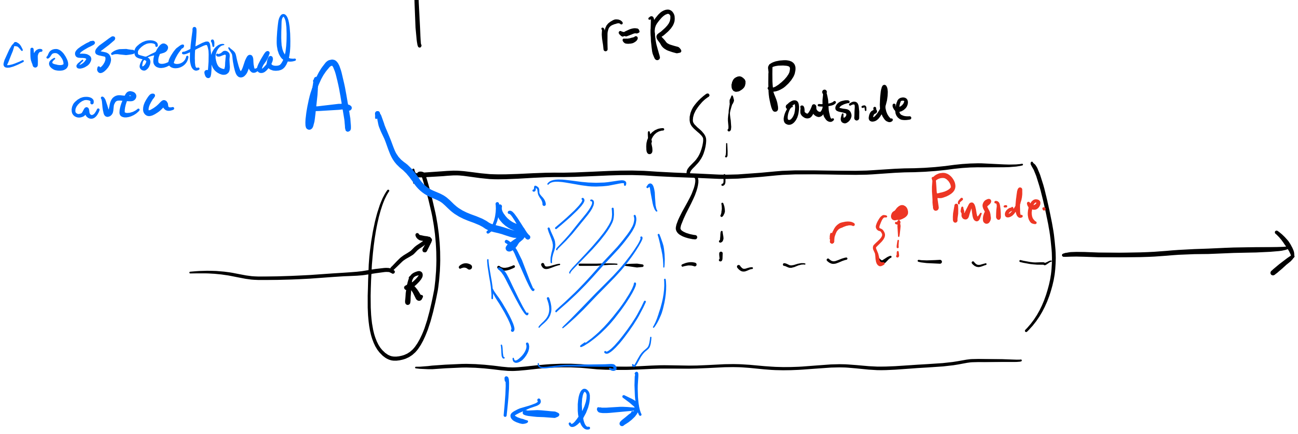
- $\vec{E}$-field inside the cylinder $(r<R)$

$$E = \frac{\rho r}{2\varepsilon_0} \qquad E \propto r$$

- $\vec{E}$-field outside the cylinder

$$E = \frac{\rho R^2}{2\varepsilon_0 r} \qquad E \propto \frac{1}{r}$$





Focus on E outside charged cylinder

$r > R$. $$E = \frac{\rho R^2}{2\varepsilon_0 r}$$

The charge density of cylinder is $\rho = \dfrac{Q}{V}$

$$\rho = \frac{Q}{A l} \Rightarrow \therefore \rho A = \frac{Q}{l} = \lambda$$

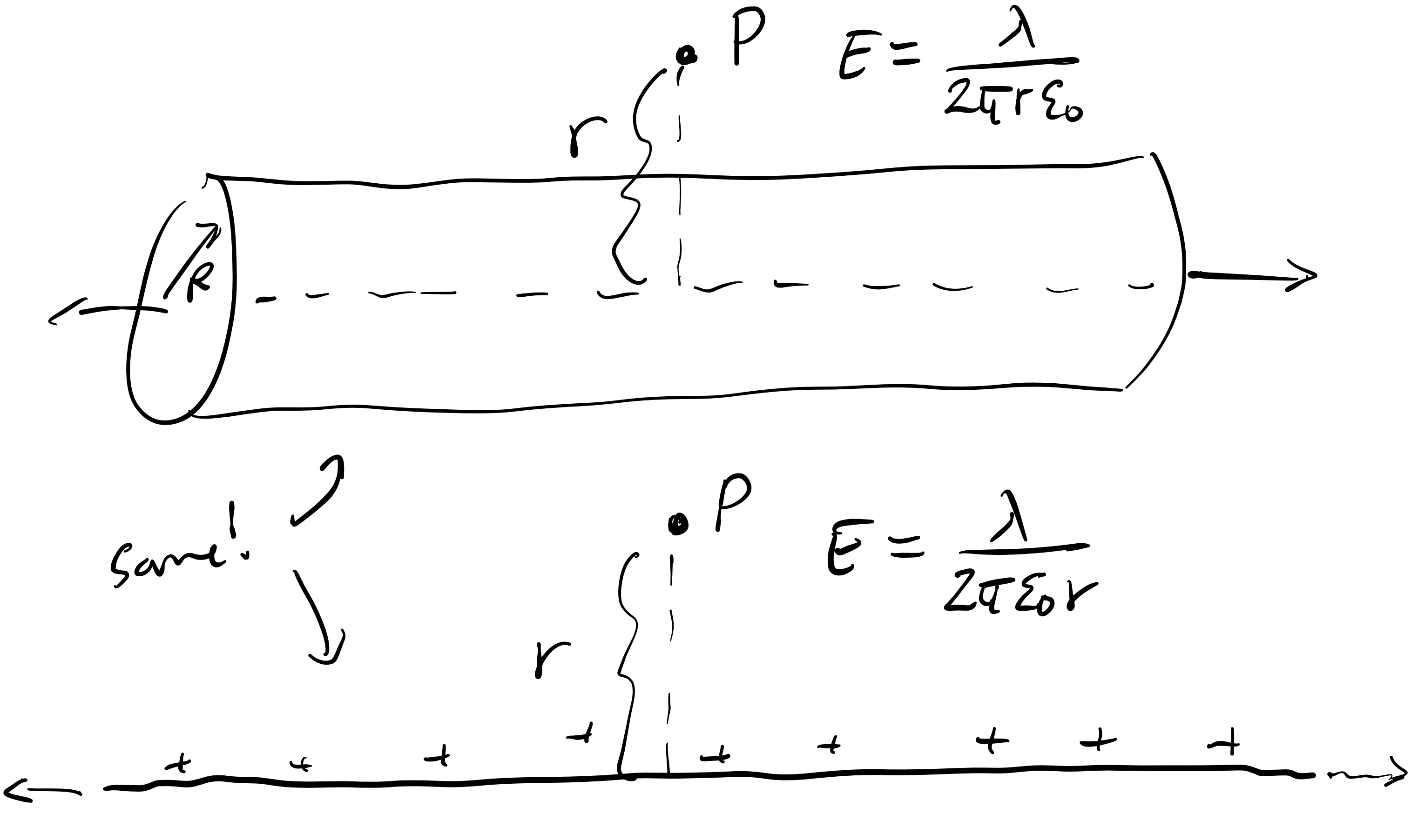
charge per unit volume

charge per unit length.

$$E = \frac{\rho R^2}{2\varepsilon_0 r}\frac{\pi}{\pi} = \frac{\rho \overbrace{(\pi R^2)}^{A}}{2\pi\varepsilon_0 r} = \frac{\rho A}{2\pi\varepsilon_0 r}$$

At a point outside charge cylinder

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$ ← same result obtained for a 1-D line of charge

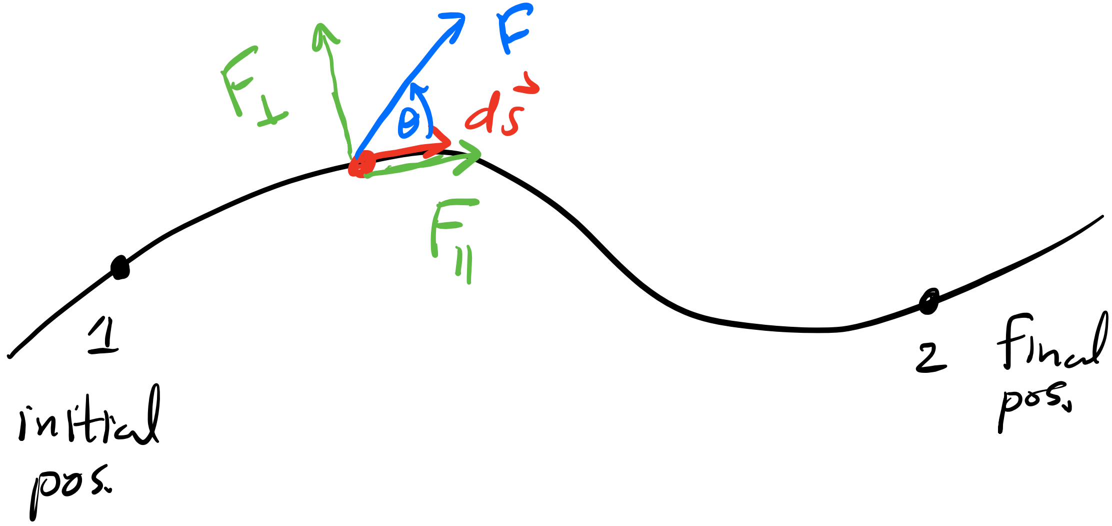


Start Chapter 7 in OSUPv2

Electric Potential & Potential Energy.

Recall the work-K.E. Theorem

$$\Delta K = \int_1^2 \vec{F} \cdot d\vec{s} = W$$



$$\vec{F}\cdot d\vec{s} = F\,ds\cos\theta$$
$$= \underbrace{(F\cos\theta)}_{F_\parallel}\,ds = F_\parallel\,ds$$

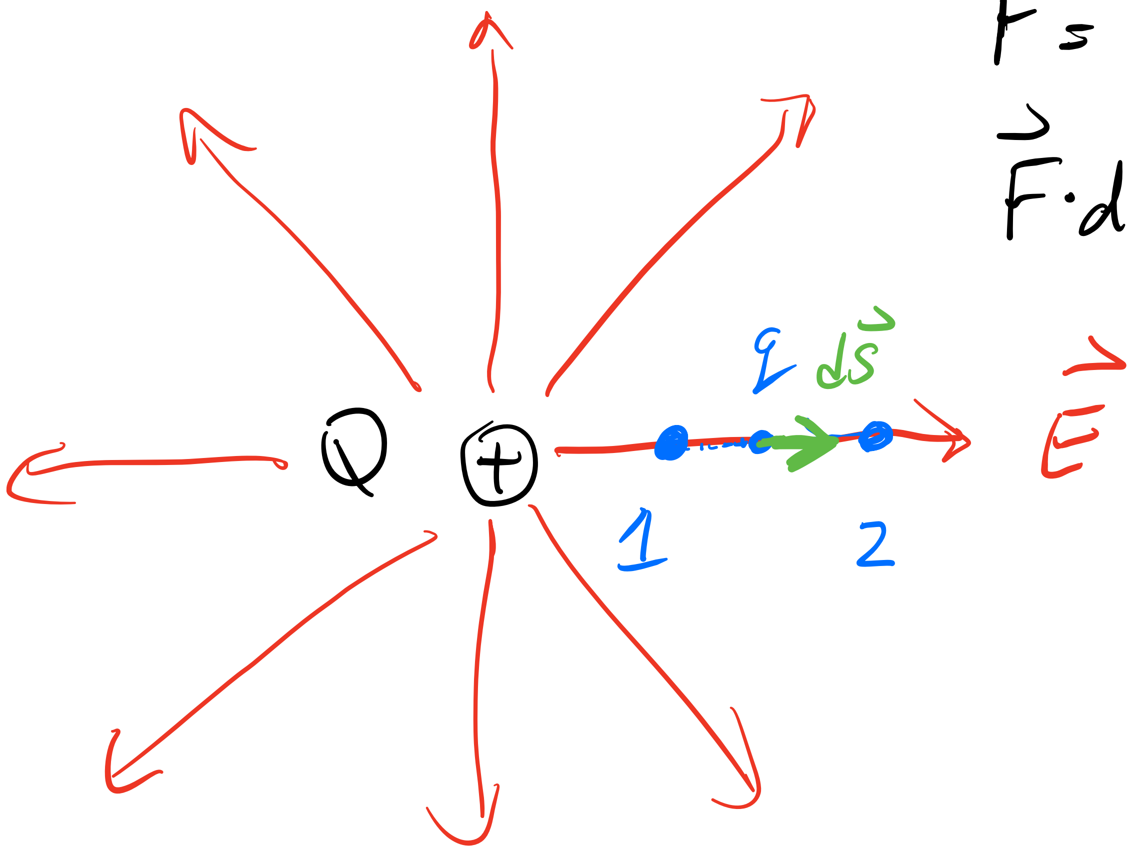
Only the component of $\vec{F}$ that is parallel to the displacement $d\vec{s}$ contributes to the work W.

Apply work-K.E. theorem to a pt charge q moving in the electric field due to another pt. charge Q.

Calculate the work for q to move along a straight line from pos 1 to pos. 2.

$$\vec{F} = q\vec{E}$$

$$\vec{F}\cdot d\vec{s} = q\vec{E}\cdot d\vec{s}$$



For our straight path $\vec{E} \parallel d\vec{s}$

s.t. $$\vec{E}\cdot d\vec{s} = |\vec{E}||d\vec{s}|\underbrace{\cos 0}_{1} = E\,ds$$

our displacement is in the radial dir'n, $ds = dr$

$$\Delta K = W = \int_1^2 \vec{F}\cdot d\vec{s} = \int_1^2 q\vec{E}\cdot d\vec{s}$$

$$= \int_1^2 q\bar{E}\,dr$$

$$= \int_1^2 q\left(\frac{k_e Q}{r^2}\right)dr$$

$$\Delta K = k_e q Q \underbrace{\int_1^2 \frac{1}{r^2}\,dr}_{-\frac{1}{r}\Big|_1^2}$$

$$\Delta K = -k_e q Q\left(\frac{1}{r}\Big|_1^2\right)$$

$$= -k_e q Q\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

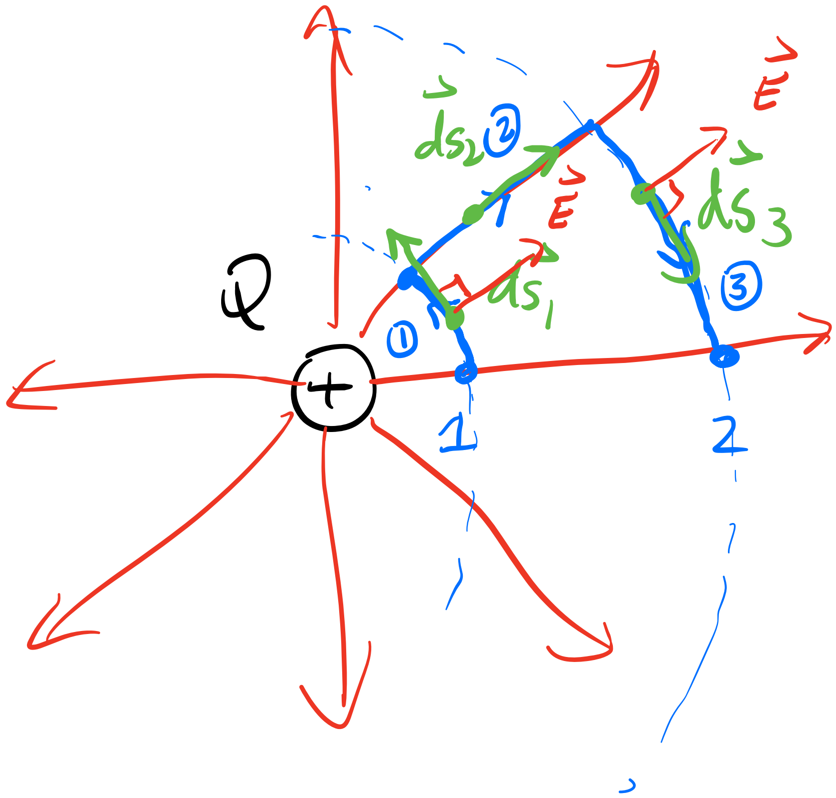
final pos.   initial pos.

If Q & q are pos., expect q to gain K.E. as it moves from 1 to 2 due to Coulomb repulsion.

$$\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \overbrace{-keqQ\underbrace{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}_{\text{less than zero}}}^{\text{positive}}$$

$\therefore v_2 > v_1$ & q has increased its speed / K.E. ✓

Repeat the work calculation using the same pts. 1 & 2, but a different path between them.



For paths ① & ③

$\vec{E} \perp d\vec{s}$

$$\therefore \vec{F}\cdot d\vec{s} = q\vec{E}\cdot d\vec{s} = 0.$$

Only path ② contributes to the work.

$$\vec{E} \parallel d\vec{s}_2$$

The work due to path ② is identical to the previous example i.e.

$$W = \cancelto{0}{\int_{①} \vec{F}\cdot d\vec{s}} + \int_{②} \vec{F}\cdot d\vec{s} + \cancelto{0}{\int_{③} \vec{F}\cdot d\vec{s}}$$

$$\Delta K = W = -k_e q Q\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Exactly the same as before!

No matter which path you choose from 1 to 2, the Work will always be same. It depends only on the starting & ending points.

The work done by the electrostatic force is path-independent.

Forces for which the work is path indep. are called conservative forces. Can define potential energies for conservative forces. We will define the electrostatic P.E. next time!