PHYS 121 Feb. 7, 2024

<u>To do</u>: - Complete HW5 by 23:59 on Friday
- No Pre-Lab #2
- If participating in Hands-On Bonus project, please email me your project proposal by 23:59 on Monday, February 12.

Recall Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}$$

For a spherical Gaussian surface surrounding a spherical charge dist'n:

$$\oint \vec{E} \cdot d\vec{A} = E\underbrace{(4\pi r^2)}_{\text{surface area of sphere of radius } r}$$

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A point charge $q = -8.0 \times 10^{-12}\ \mathrm{C}$ is placed at the centre of a spherical conducting shell of inner radius $3.6\ \mathrm{cm}$ and outer radius $3.9\ \mathrm{cm}$. The electric field just above the surface of the conductor is directed radially outward and has magnitude $9.5\ \mathrm{N/C}$.

Part 1

What is the charge density on the inner surface of the shell?

$\sigma_{\text{inner}} =$ number (rtol=0.03, atol=0) $\mathrm{C/m^2}$

Part 2

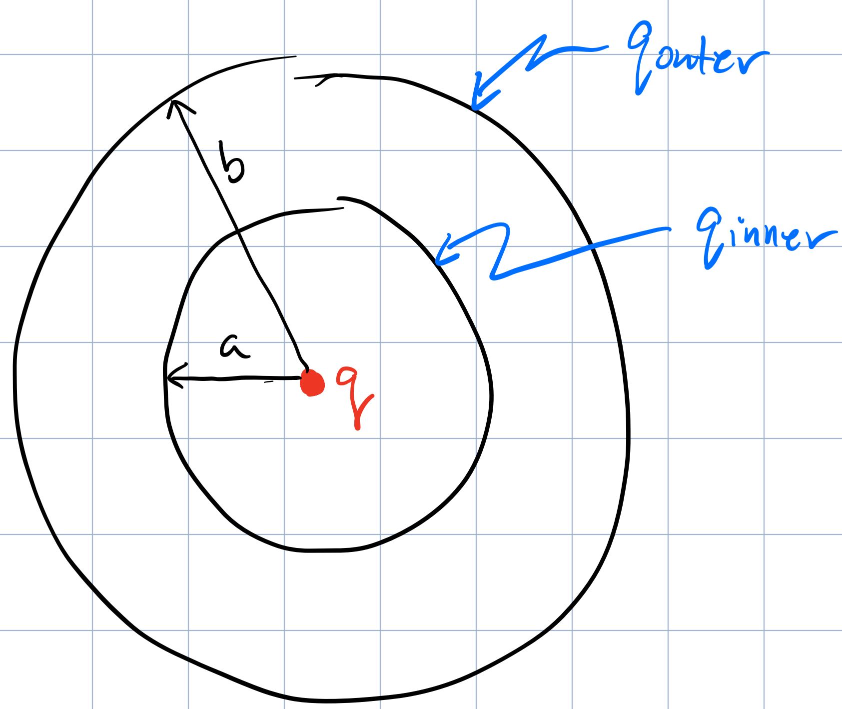
What is the charge density on the outer surface of the shell?

$\sigma_{\text{outer}} =$ number (rtol=0.03, atol=0) $\mathrm{C/m^2}$

Part 3

What is the net charge on the conductor?

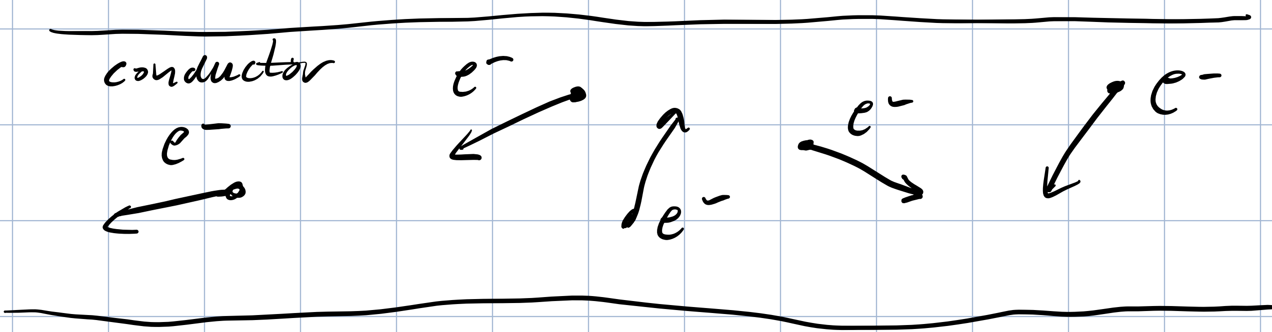
$Q =$ number (rtol=0.03, atol=0) C



Conductors in Electric Fields.

Some electrons in a conductors are free to move around inside the boby of the conductor.

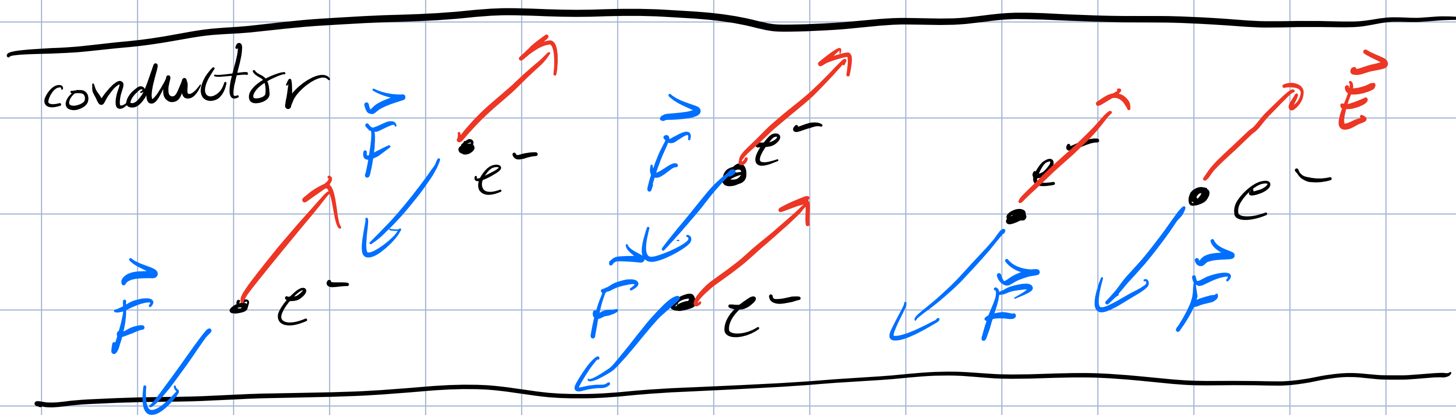
→ called conduction electrons.



A conductor is said to be in equil. when there is no net flow of charge in any particular dir'n.

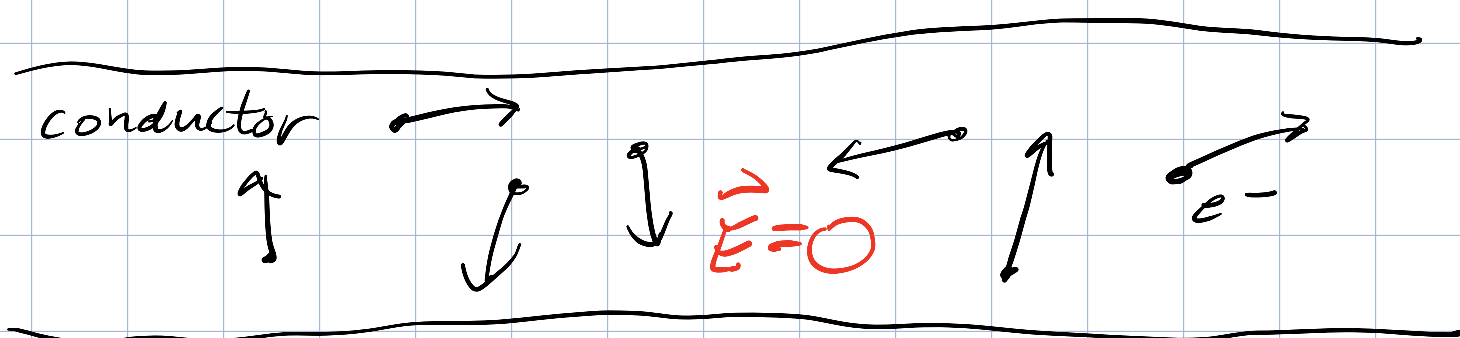
Charge can still move around randomly, but in equil., there cannot be any net motion in any single dir'n.

① For a conductor in equil., the electric field inside the body is zero.

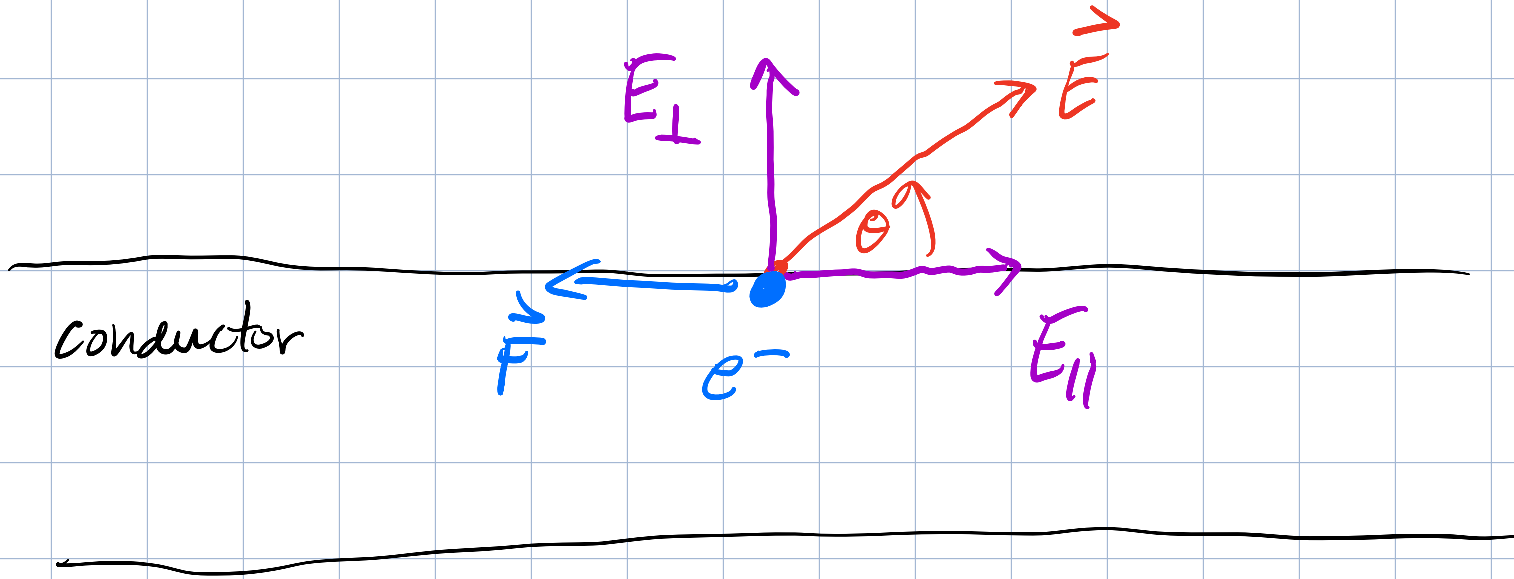


If $\vec{E} \neq 0$ inside conductor (as shown above), it exerts a force $\vec{F} = -e\vec{E}$ on the mobile charges.

This force would cause conduction electrons to all accelerate/move in the same dir'n.
$\therefore$ the conductor cannot be in equil.

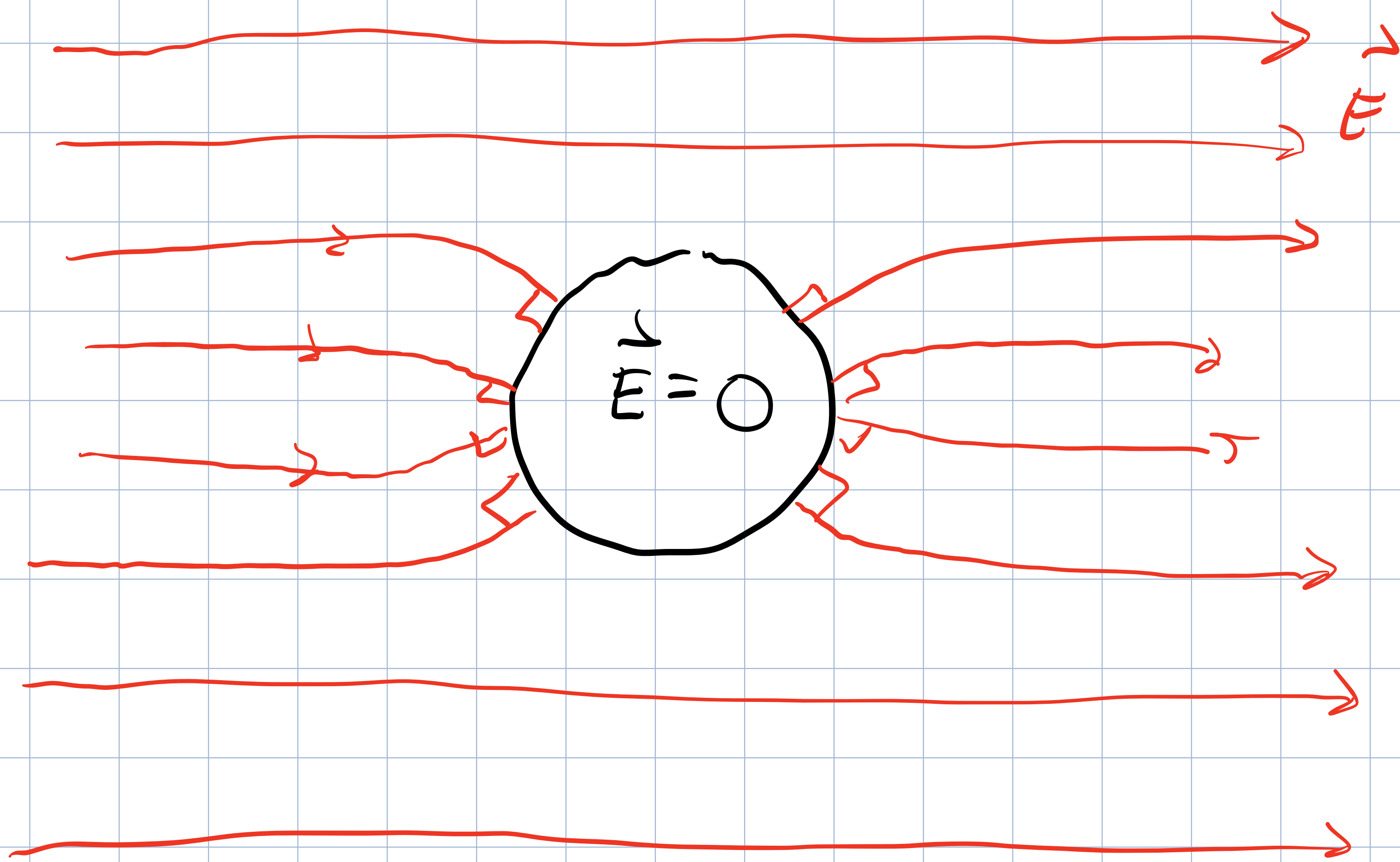


(2) For a conductor in equil., the electric field just outside in $\perp$ to conductor's surface.



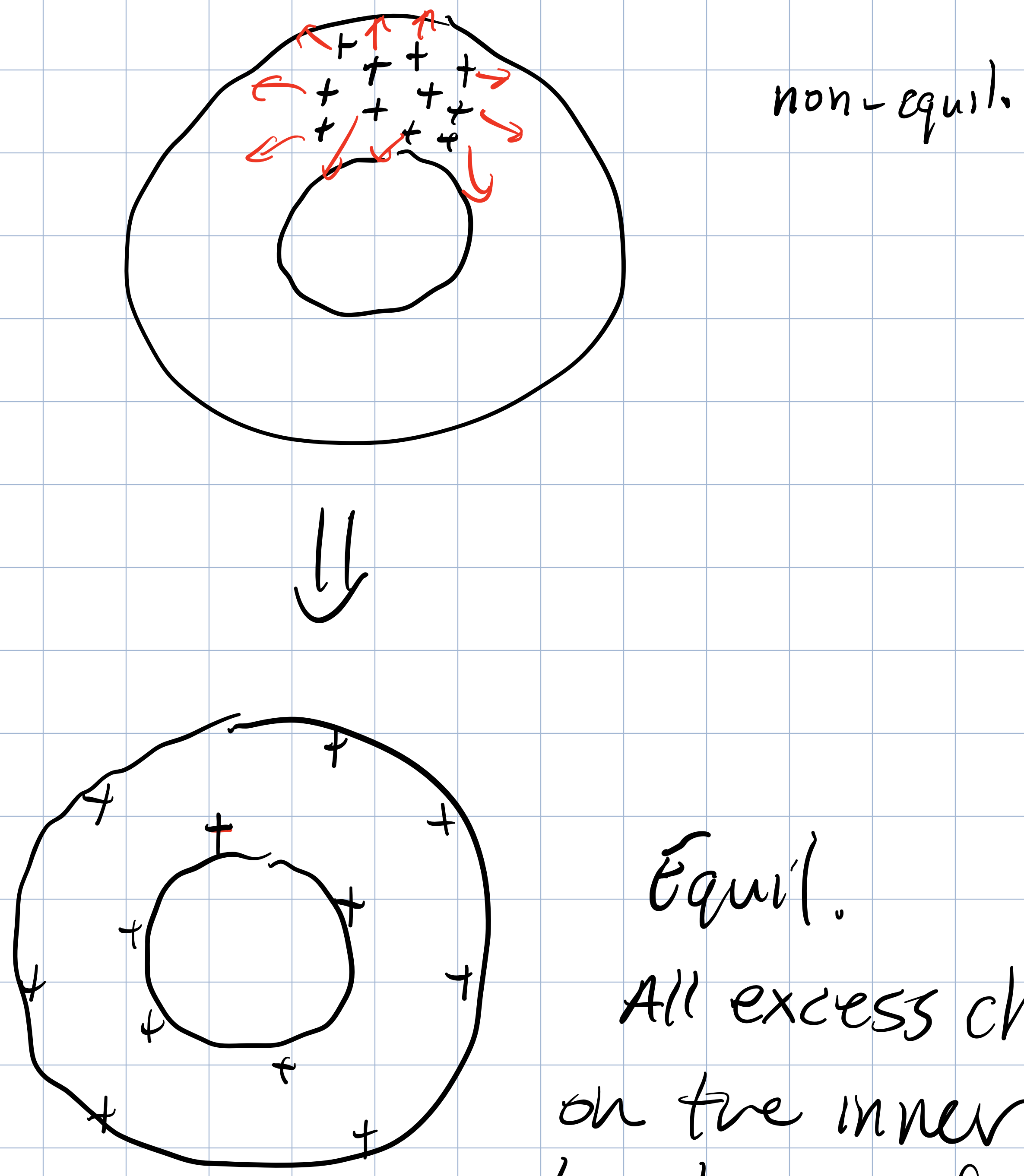
If there is an $\vec{E}$-field outside conductor that is <u>not</u> $\perp$ to surface, it cause e^- near the surface to flow in the opposite dir'n of $E_{\parallel}$ & $\therefore$ conductor is <u>not</u> in equil. Require $E_{\parallel} = 0$ for conductors in equil.

Eg. Consider a conductor in equil. placed in a uniform electric field.



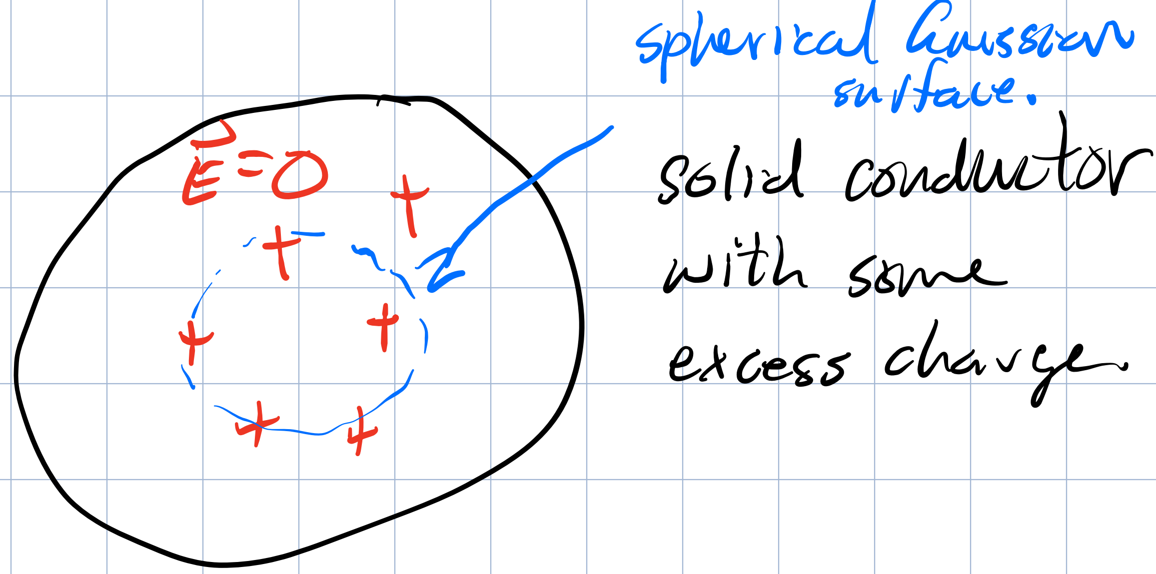
③ All excess charge on a conductor resides at the conductor's surface.

If place excess charge on a conductor those charges repel one another & migrate to the surfaces (inner & outer surface) due to Coulomb repulsion between like charges.



All excess charge on the inner & outer surfaces.

Another argument using Gauss's law



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0} = 0$$

$\vec{E}=0$ for conductor.

If $\vec{E}=0$, then $\oint \vec{E}\cdot d\vec{A} = 0$

∴ we require $q_{encl} = 0$

∴ Conductor in equil. must have all excess charge a surface.

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Part 1

What is the charge density on the inner surface of the shell?

$\sigma_{\text{inner}} =$ number (rtol=0.03, atol=0) $\mathrm{C/m^2}$?

Part 2

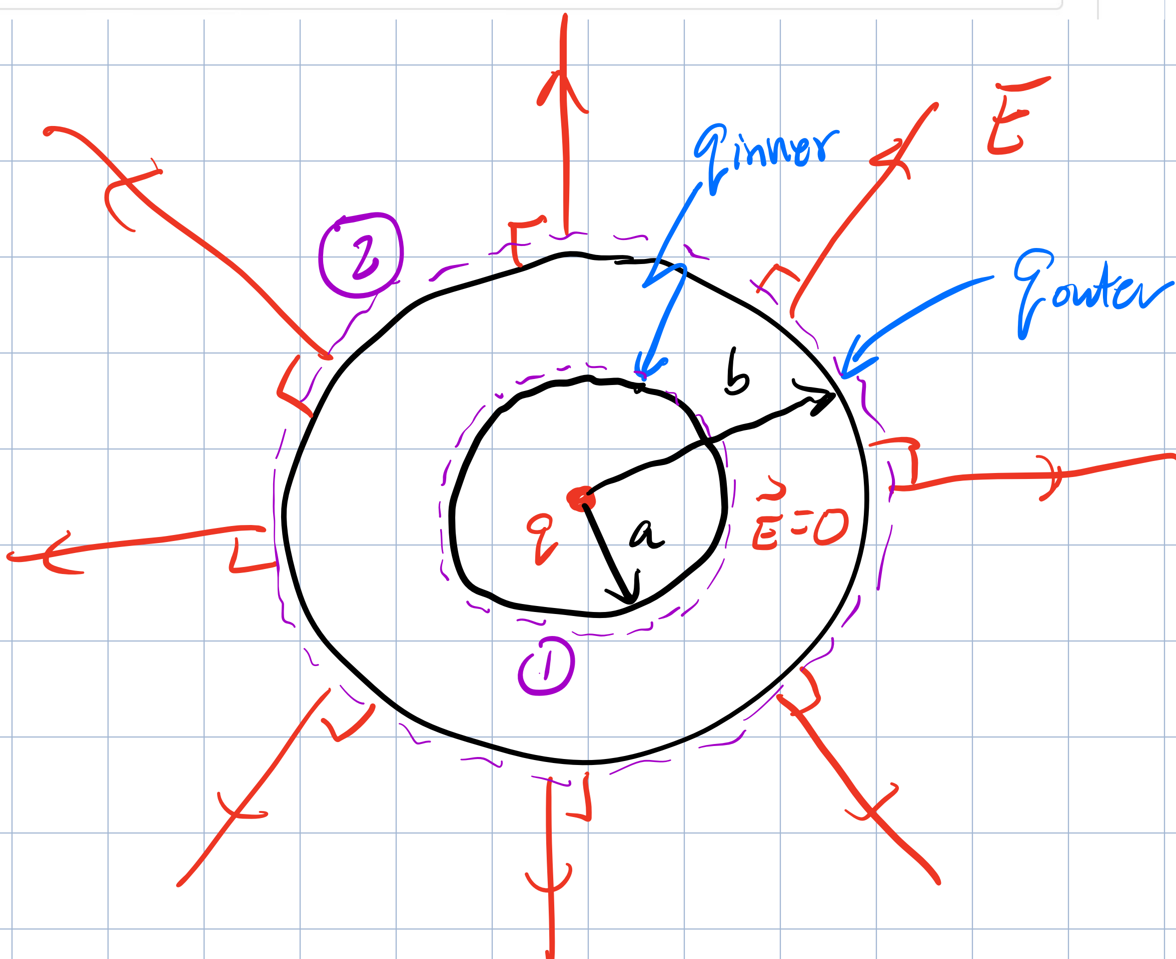
What is the charge density on the outer surface of the shell?

$\sigma_{\text{outer}} =$ number (rtol=0.03, atol=0) $\mathrm{C/m^2}$?

Part 3

What is the net charge on the conductor?

$Q =$ number (rtol=0.03, atol=0) C ?



Find the charge density at inner surface. (Part 1).

Place Gaussian surface ① just around inner wall of shell.

$$\oint_{①} \vec{E} \cdot d\vec{A} = q_{encl}.$$

$\vec{E} = 0$ b/c surface is inside conductor.

$$\therefore \oint_{①} \cancel{\vec{E}}^{\,0} \cdot d\vec{A} = 0$$

$$q_{encl} = q + q_{inner}$$

$\therefore$ Gauss's law requires

$$0 = \frac{q + q_{inner}}{\varepsilon_0}$$

$$\Rightarrow \boxed{q_{inner} = -q}$$

$$\boxed{\sigma_{inner} = \frac{q_{inner}}{A_{inner}} = \frac{-q}{4\pi a^2}}$$

Part 2: Find charge density at outer surface.

Place a second Gaussian surface ② just outside the spherical shell.

$$\oint_{②} \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}$$

$$\underbrace{E(4\pi b^2)}_{} = \frac{\cancel{q} + \cancel{\overset{-q}{\widetilde{q_{inner}}}} + q_{outer}}{\varepsilon_0}$$

$$\therefore \boxed{q_{outer} = \varepsilon_0 E (4\pi b^2)}$$

$$\boxed{\sigma_{outer} = \frac{q_{outer}}{4\pi b^2} = \varepsilon_0 E}$$

Part 3: Net charge on conductor is

$$\boxed{\begin{aligned} q_{net} &= q_{inner} + q_{outer} \\ &= -q + \varepsilon_0 E (4\pi b^2) \end{aligned}}$$