PHYS 121 Feb. 12, 2024

To do: ✓ Complete HW6 by 23:59 Friday

✓ Complete Pre-Lab #3 before your lab

✓ If participating in Hands-On Bonus project, please email me your project proposal by 23:59 today

Last Time:

- Calculating change in potential from $\vec{E}$.

$$\Delta V = -\int_1^2 \vec{E} \cdot d\vec{s}$$

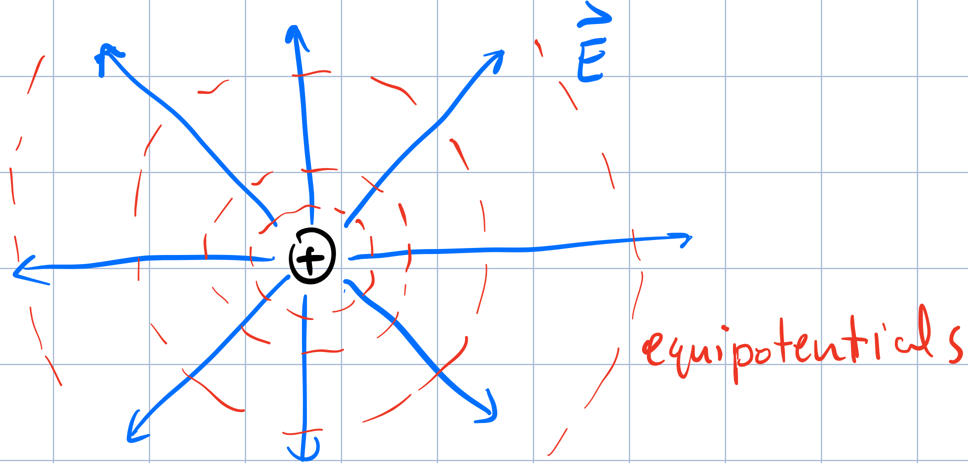
$[\Delta V] = [E] \cdot [ds]$

$[\Delta V] = \frac{N}{C} m = \frac{J}{C} = 1V$ (volt)

If $d\vec{s} \perp \vec{E}$, $\vec{E} \cdot d\vec{s} = 0 \Rightarrow \Delta V = 0$

$\therefore$ V is const. if move $\perp$ to $\vec{E}$

$\boxed{\Rightarrow \text{Equipotentials are } \perp \text{ to } \vec{E}\text{-field lines}}$



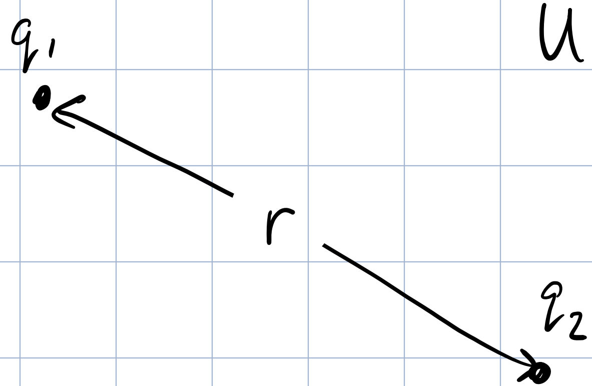
Calculating $\vec{E}$ from V.

$$E_s = -\frac{dV}{ds}$$

component of $\vec{E}$ in s-dir'n.

$$\vec{E} = \underbrace{-\frac{dV}{dx}}_{E_x}\,\hat{\imath} \underbrace{-\frac{dV}{dy}}_{E_y}\,\hat{\jmath}$$

P.E. of a pair of pt. charges

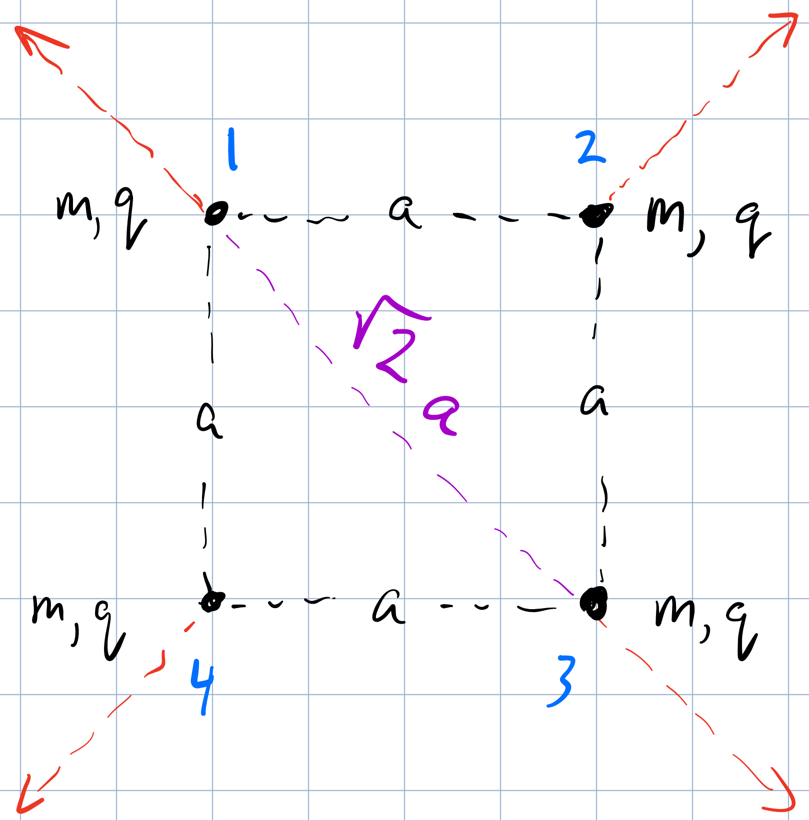


$$U = \frac{k_e q_1 q_2}{r}$$

Electric Potential of a pt. charge

$$V = \frac{k_e q}{r}$$

Eg.



Four identical pt. charges w/ charge q & mass m are at rest at the corners of a square of sides length a. At $t=0$, the particles are released from rest & move away from each other due to Coulomb repulsion. How fast are the particles moving once they are infinitely far away from one another?

(a) Find initial P.E. of the system

$$U_{net} = U_{12} + U_{13} + U_{14}$$
$$+ U_{23} + U_{24}$$
$$+ U_{34}$$

Have 6 pairs of charges. Find the P.E. of each pair & sum to get U_{net}.

$$U_{12} = \frac{k_e q_1 q_2}{r_{12}} = \frac{k_e q^2}{a}$$

$$U_{12} = U_{23} = U_{34} = U_{14}$$

$$U_{13} = U_{24} = \frac{k_e q^2}{\sqrt{2} a}$$

$$U_{net} = 4\left(\frac{k_e q^2}{a}\right) + 2\left(\frac{k_e q^2}{\sqrt{2}\,a}\right)$$

$$\boxed{= \frac{k_e q^2}{a}\left[4 + \sqrt{2}\right]} = U_i$$

Initial mechanical energy of system is

$$E_i = U_i + \cancel{K_i}^{\,\to 0} = \frac{k_e q^2}{a}\left[4 + \sqrt{2}\right]$$

b/c particles @ rest

(b) Find the final mech. energy once particles are infinitely far apart.

B/c all particles are identical, they reach the same final speed v.

$$K_f = 4\left(\frac{1}{2}mv^2\right) = 2mv^2$$

$U_f = 0$ b/c $U \propto \frac{1}{r}$ & $r \to \infty$.

$E_f = K_f + U_f = 2mV^2$

(c) Find final speed.

Conservation of mech. energy:

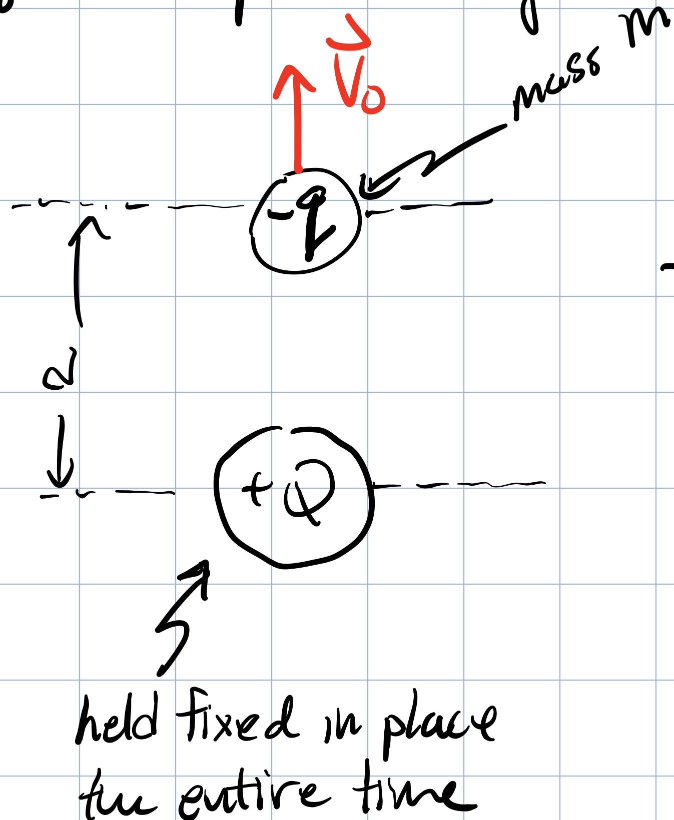
$$E_i = E_f$$

$$\frac{k_e q^2}{a}\left[4 + \sqrt{2}\right] = 2mV^2$$

$$V^2 = \frac{k_e q^2}{ma}\left[2 + \frac{1}{\sqrt{2}}\right]$$

$$\boxed{V = \sqrt{\frac{k_e q^2}{ma}\left[2 + \frac{1}{\sqrt{2}}\right]}}$$

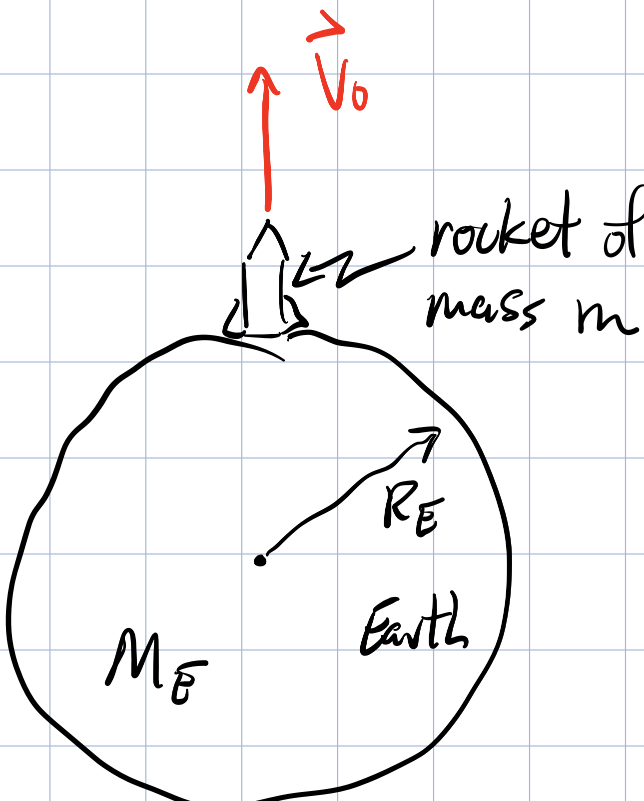
Eg. Escape velocity.



How fast must the initial speed v_0 be s.t. $-q$ & $+Q$ never come together? (What is speed v_0 required s.t. $-q$ escapes the pull of $+Q$?)

$$U = \frac{-k_e q Q}{r}$$

Parallel Problem:



What is the required speed v_0 s.t. rocket never returns to Earth?

Find the escape velocity.

$$U_g = \frac{-G M_E m}{r}$$

charges	rocket
initial	
$U_i = -\frac{k_e q Q}{d}$	$U_{gi} = -\frac{G M_E m}{R_E}$
$K_i = \frac{1}{2} m v_0^2$	$K_i = \frac{1}{2} m v_0^2$
final $U_f = 0 \quad (r \to \infty)$	$U_f = 0 \quad (r \to \infty)$
To ensure that charge/rocket never stops & turns around, require $v=0$ only when $r \to \infty$.	
$K_f = 0$	$K_f = 0$
conserve mech. energy.	
$U_i + K_i = \cancel{U_f}^{\to 0} + \cancel{K_f}^{\to 0}$	
$-U_i = K_i$	$-U_i = K_i$
$\therefore \frac{k_e q Q}{d} = \frac{1}{2} m v_0^2$	$\frac{G M_E \cancel{m}}{R_E} = \frac{1}{2} \cancel{m} v_0^2$

Solve for V_0, the escape velocity.

$$\boxed{V_0 = \sqrt{\frac{2k_e qQ}{md}}}$$

$$\boxed{V_0 = \sqrt{\frac{2GM_E}{R_E}}}$$

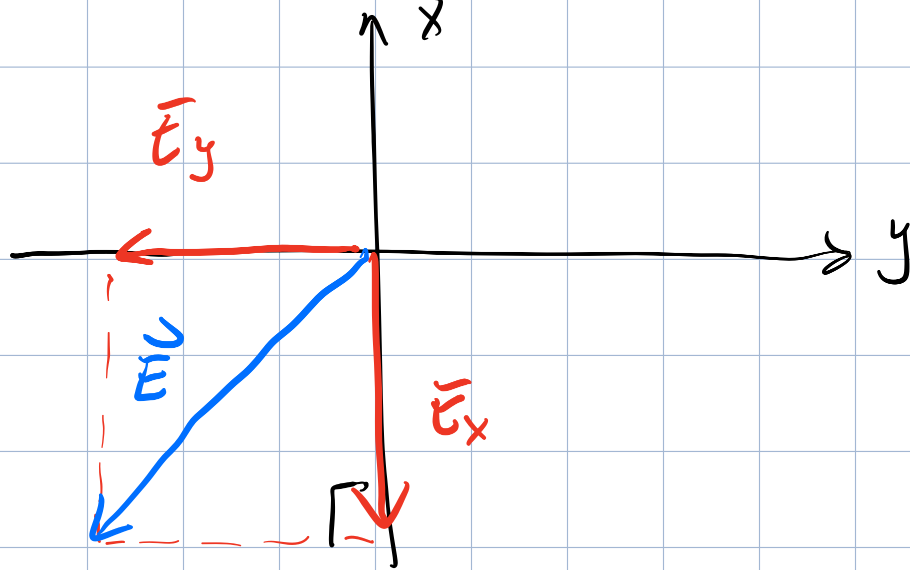
Eg. Find the $\vec{E}$-field vector & magnitude if $V = 2x^2y + y^3\pi$

Recall: $E_s = -\frac{dV}{ds}$

Find x-component $E_x = -\frac{dV}{dx} = -4xy$

y-component $E_y = -\frac{dV}{dy} = -2x^2 - 3\pi y^2$

$$\boxed{\begin{aligned}\vec{E} &= E_x\hat{i} + E_y\hat{j} \\ &= -\left[4xy\,\hat{i} + (2x^2 + 3\pi y^2)\,\hat{j}\right]\end{aligned}}$$



$$|\vec{E}|^2 = E_x^2 + E_y^2$$

$$\boxed{|\vec{E}| = \sqrt{(-4xy)^2 + (-2x^2 - 3\pi y^2)^2}}$$

$$\vec{E} = (E_x \hat{\imath} + E_y \hat{\jmath}) \cdot (E_x \hat{\imath} + E_y \hat{\jmath})$$

$$= E_x^2 \underbrace{(\hat{\imath} \cdot \hat{\imath})}_{1} + E_y^2 \underbrace{(\hat{\jmath} \cdot \hat{\jmath})}_{1}$$