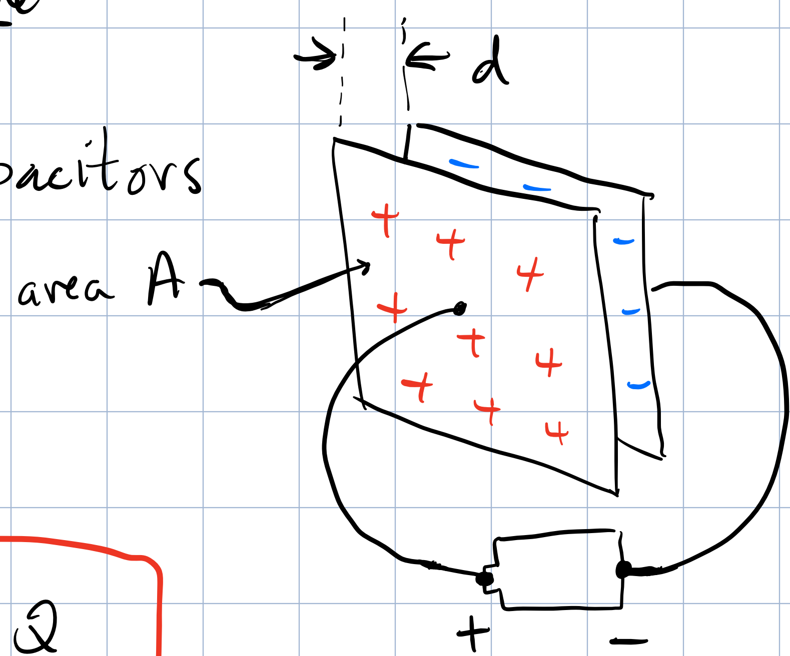
PHYS 121 Feb. 16, 2024

<u>To do</u>:

- Complete HW6 by 23:59 today
- Reading week next week ⇒ no classes
- Complete Pre-Lab #4 before Lab #4
- Midterm Wed. Feb. 28 (in person EME 0050)
- No tutorials the week of the Midterm

<u>Last Time</u>:

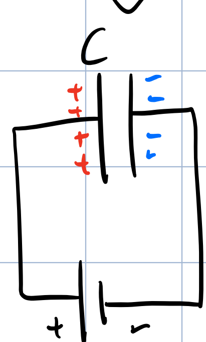
Capacitors



$$\boxed{C = \frac{Q}{|\Delta V|}}$$

For a parallel-plate capacitor

$$\boxed{C = \varepsilon_0 \frac{A}{d}}$$



Eg. A parallel plate cap. is connected to a battery of voltage ΔV. The charge on the plates is $\pm Q_1$.

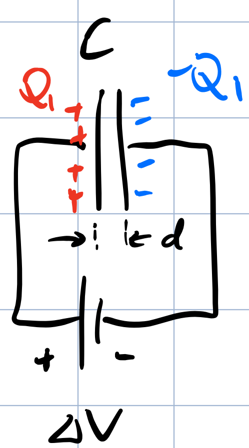
After disconnecting the battery, the capacitor plates are pulled appart s.t.

$d \to 3d$.

→ (a) What is the new charge on the plates?

→ (b) What is the new potential difference across the plates?

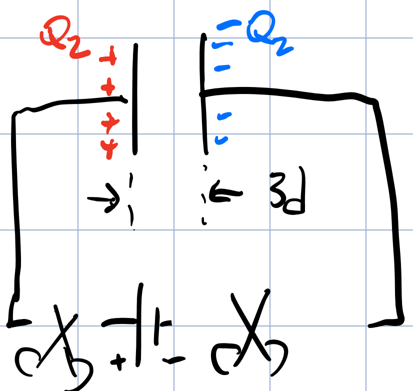
Initial



$$C = \frac{Q}{|\Delta V|}$$

$$Q_1 = C|\Delta V| = \varepsilon_0 \frac{A}{d} |\Delta V|$$

(a) Final



Since there is no closed circuit, the charge cannot flow

$\therefore$ Q remains constant.

$$\boxed{Q_1 = Q_2}$$

(b) $C = \dfrac{Q}{|\Delta V|}$ $\quad \therefore \quad |\Delta V| = \dfrac{Q}{C}$

Initial

$Q_1 = Q$

$$|\Delta V_1| = \frac{Q}{C_1} = \frac{Q}{\varepsilon_0 \frac{A}{d}}$$

$$|\Delta V_1| = \frac{Qd}{\varepsilon_0 A}$$

Final

$Q_2 = Q$

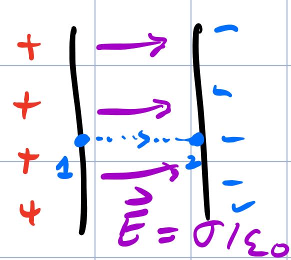
$$|\Delta V_2| = \frac{Q}{C_2} = \frac{Q}{\varepsilon_0 \frac{A}{3d}}$$

$$|\Delta V_2| = 3\underbrace{\left(\frac{Qd}{\varepsilon_0 A}\right)}_{|\Delta V_1|}$$

$$\boxed{\therefore \ |\Delta V_2| = 3|\Delta V_1|}$$

Alternative Approach...

If Q is const., then we know that electric field between the plates is given by $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{\varepsilon_0 A}$



$\therefore$ In this problem, $\vec{E}$ remains const. when plates are pulled apart.

Recall that $|\Delta V| = \int_1^2 \vec{E} \cdot d\vec{l}$

For our parallel plate cap., $\int \vec{E} \cdot d\vec{l} = E l$

<u>initial</u>

$l = d \quad \therefore |\Delta V_1| = E d$

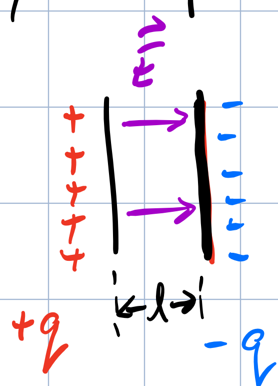
final

$l = 3d \quad |\Delta V_2| = 3Ed$

$= 3|\Delta V_1|$

Energy Stored in a Charge Capacitor

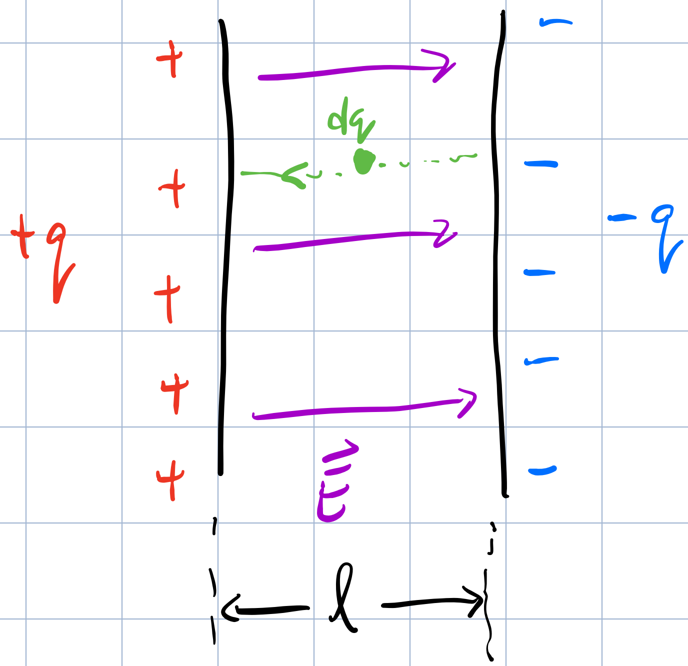
Start w/ a capacitor charged to $\pm q$.



Know $E = \frac{\sigma}{\varepsilon_0}$

$= \frac{q}{\varepsilon_0 A}$

Also know $\Delta V = E l = \frac{q l}{\varepsilon_0 A}$



Consider moving small pos. charge dq from neg. plate to pos. plate through electric field $\vec{E}$ / a potential difference V.

The energy cost is $dU = dq\,\Delta V$

$$= \left(\frac{q\ell}{\varepsilon_0 A}\right) dq$$

$$dU = \left(\frac{\ell}{\varepsilon_0 A}\right) q \boxed{dq}$$

small charge.

The total energy required to charge the capacitor is the sum of all dU starting from an uncharge cap. & ending w/ fully-charged cap. Final charge is Q.

$$U = \sum dU = \sum_{q_i=0}^{q_f=Q} \left(\frac{\ell}{\varepsilon_0 A}\right) q\,dq$$

Factor constants outside sum

$$U = \left(\frac{l}{\varepsilon_0 A}\right) \sum_{q_i=0}^{q_f=Q} q\,dq$$

Take the limit that $dq \to 0$, the sum becomes an integral.

$$U = \left(\frac{l}{\varepsilon_0 A}\right) \int_{q_i=0}^{q_f=Q} q\,dq$$

$$= \left(\frac{l}{\varepsilon_0 A}\right) \frac{q^2}{2}\Bigg|_0^Q = \left(\frac{l}{\varepsilon_0 A}\right) \frac{1}{2}\left(Q^2 - 0^2\right)$$

$$U = \frac{1}{2} \frac{l\, Q^2}{\varepsilon_0 A} \qquad \frac{l}{\varepsilon_0 A} \to \frac{1}{C}$$

Recall that for our parallel plate cap.

$$C = \varepsilon_0 \frac{A}{l}$$

$$\boxed{\therefore U = \frac{Q^2}{2C}}$$

Energy stored by a cap. charged to $\pm Q$.

Substitute $C = \frac{Q}{|\Delta V|}$

$$U = \frac{Q^{\cancel{2}}}{2\left(\frac{\cancel{Q}}{|\Delta V|}\right)} = \boxed{\frac{1}{2} Q |\Delta V|}$$

Let's now express $Q = C|\Delta V|$

$$\boxed{U = \frac{1}{2} C |\Delta V|^2}$$

Three equivalent ways of expressing energy stored by a cap:

$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2} Q |\Delta V| = \frac{1}{2} C |\Delta V|^2}$$