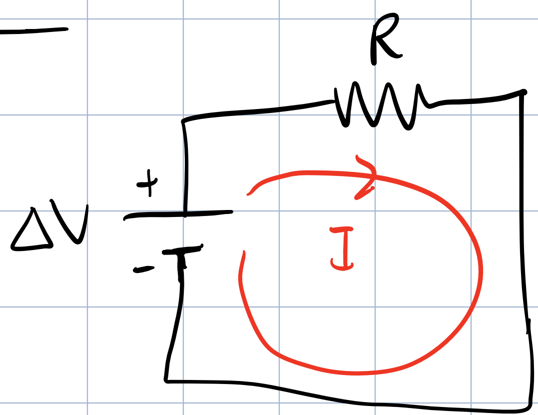
PHYS 121 Mar. 1, 2024

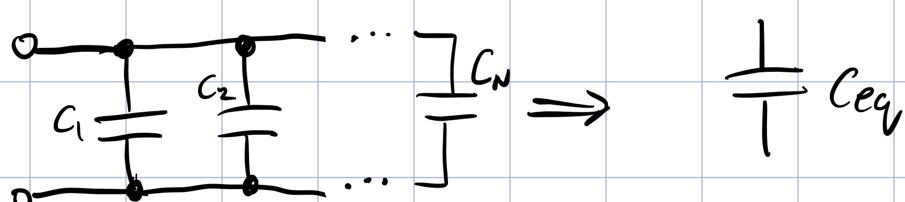
To do: - Complete HW7 by 23:59 today
- No Pre-Lab #5
- Tutorials resume next week.

Last Time:



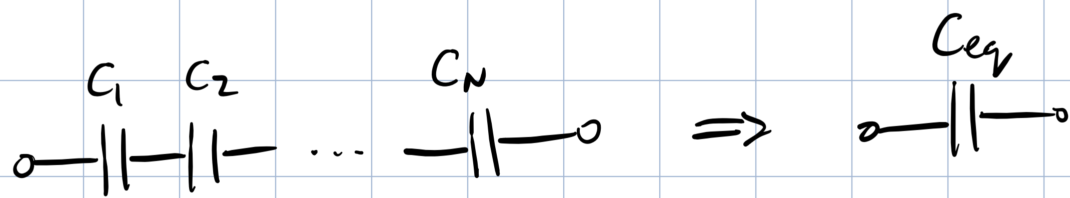
$$R = \frac{\Delta V}{I}$$

Capacitors in Parallel:



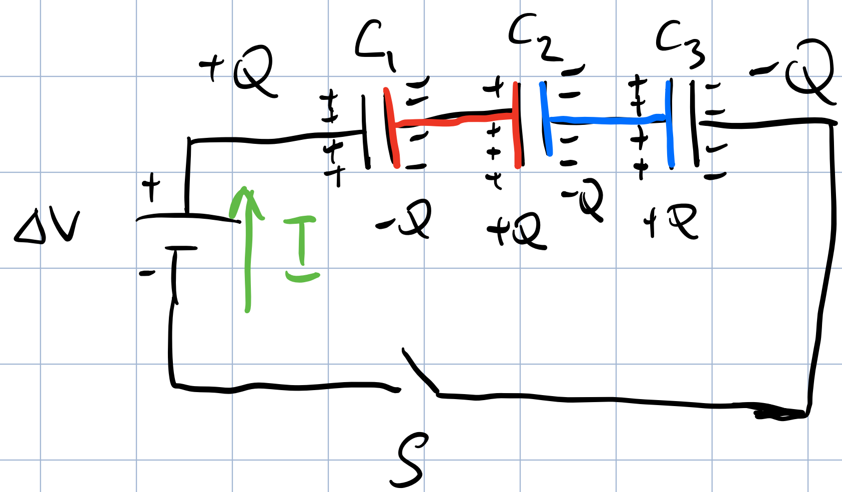
$$C_{eq} = \sum_{i=1}^{N} C_i = C_1 + C_2 + \dots + C_N$$

Capacitors in Series:



$$\frac{1}{C_{eq}} = \sum_{i=1}^{N} \frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$$

Proof: Capacitors in Series.



- Start w/ uncharged capacitors & switch S open.

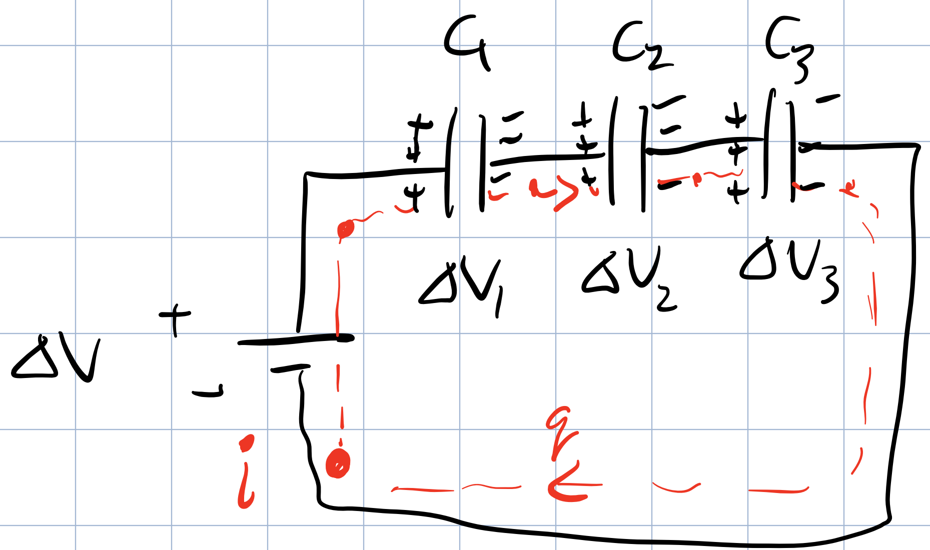
- Conductors highlighted red & blue are neutral & isolated from rest of circuit. So, when switch S is closed, the red & blue conductors <u>must</u> remain overall neutral.

- When switch is closed, battery causes charge to transfer from right plate of C_3 to left plate of C_1

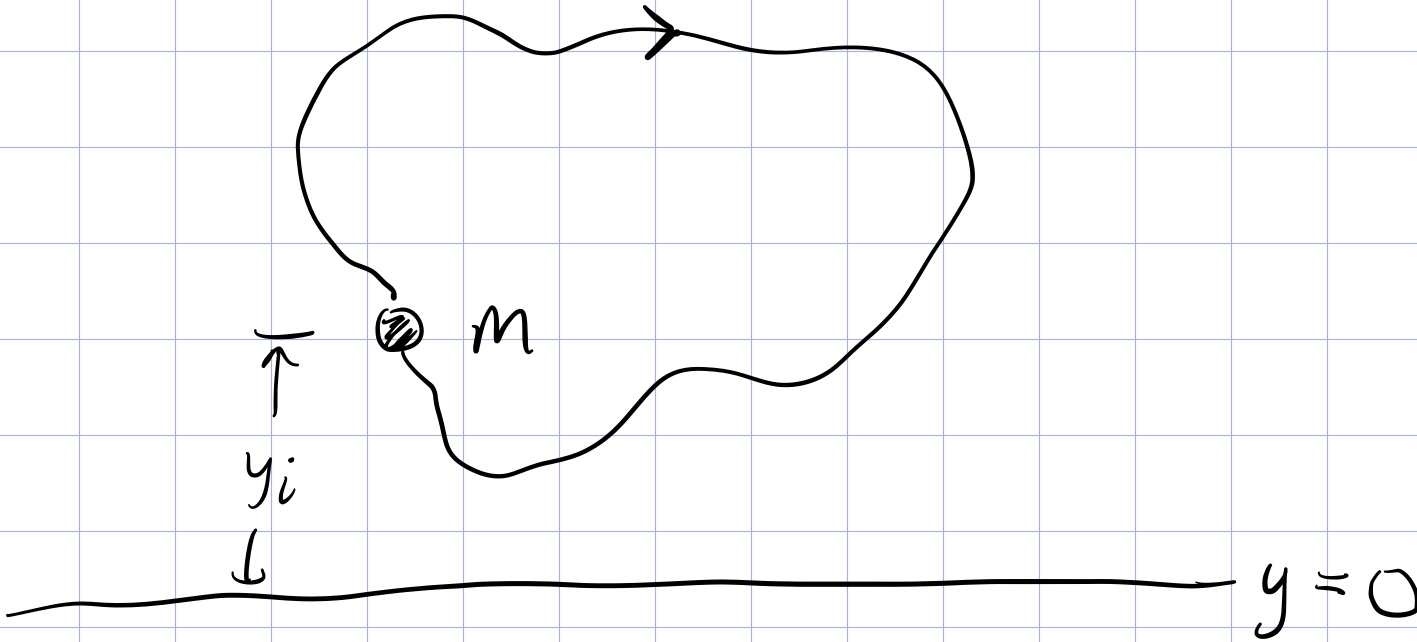
- The nuetral conductors (red & blue) become polarized (separation of charge).

⇒ Each of the series capacitors carry the same charge:

$$Q_1 = Q_2 = Q_3 \equiv Q$$



Aside: Gravitational P.E.



If we move mass m through a loop in a constant gravitation field, then net change in gravitational P.E. is zero.

$$\Delta U_g = mg\,\Delta y$$

For a loop, $\Delta y = 0 \quad \therefore \Delta U_g = 0$

Now consider a charge q moving around a circuit.

Every time the circuit has a change in potential/voltage ΔV, the corresponding change in P.E. of the charge is $\Delta U = q\Delta V$.

If we start in lower-left corner of circuit & go around clockwise, track the changes in P.E. of q:

1. Cross battery from neg to pos. terminal. Gain voltage ΔV & P.E. $\underline{\underline{q\Delta V}}$.

2. Cross C_1 from pos. to neg. plate & loss voltage $-\Delta V_1$ & P.E. $-q\Delta V_1$

3. Cross C_2 loss P.E. $-q\,\Delta V_2$

   " C_3 " " $-q\Delta V_3$

4. We're now back @ starting point. Net change in P.E. of q must be zero (conservation of energy).

$$\Delta U_{net} = \underbrace{q\Delta V}_{\text{battery}} - \underbrace{q\Delta V_1}_{C_1} - \underbrace{q\Delta V_2}_{C_2} - \underbrace{q\Delta V_3}_{C_3} = 0$$

Divide by q:

$$\Delta V - \Delta V_1 - \Delta V_2 - \Delta V_3 = 0$$

$$\boxed{\begin{array}{c}\text{In general, for any closed loop in a circuit, the net change in voltage}\\ \text{around the loop must be zero.}\\ \\ \text{Kirchhoff Voltage Loop Rule}\end{array}}$$

1 $\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$ For series cap. example.

For capacitors, we know $C = \dfrac{Q}{\Delta V}$

$$\Downarrow$$

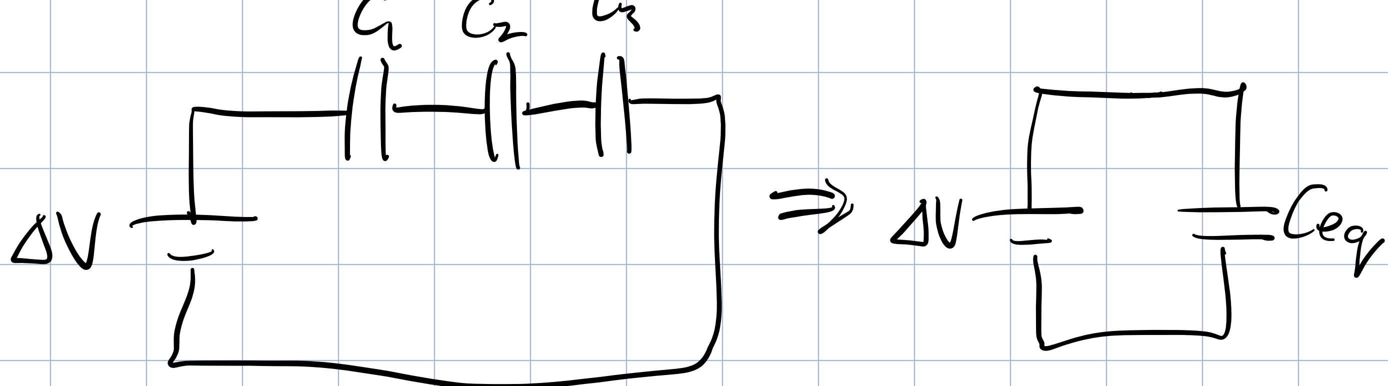
$\Delta V = \dfrac{Q}{C}$ for a cap.

$$\Delta V_1 = \frac{Q_1}{C_1} \qquad \Delta V_2 = \frac{Q_2}{C_2} \qquad \Delta V_3 = \frac{Q_3}{C_3}$$

Subbing into ①:

$$\Delta V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

Equivalent Capacitor



Require $Q_{eq} = Q_1 = Q_2 = Q_3 \equiv Q$

$\therefore Q_{eq} = C_{eq} \Delta V$

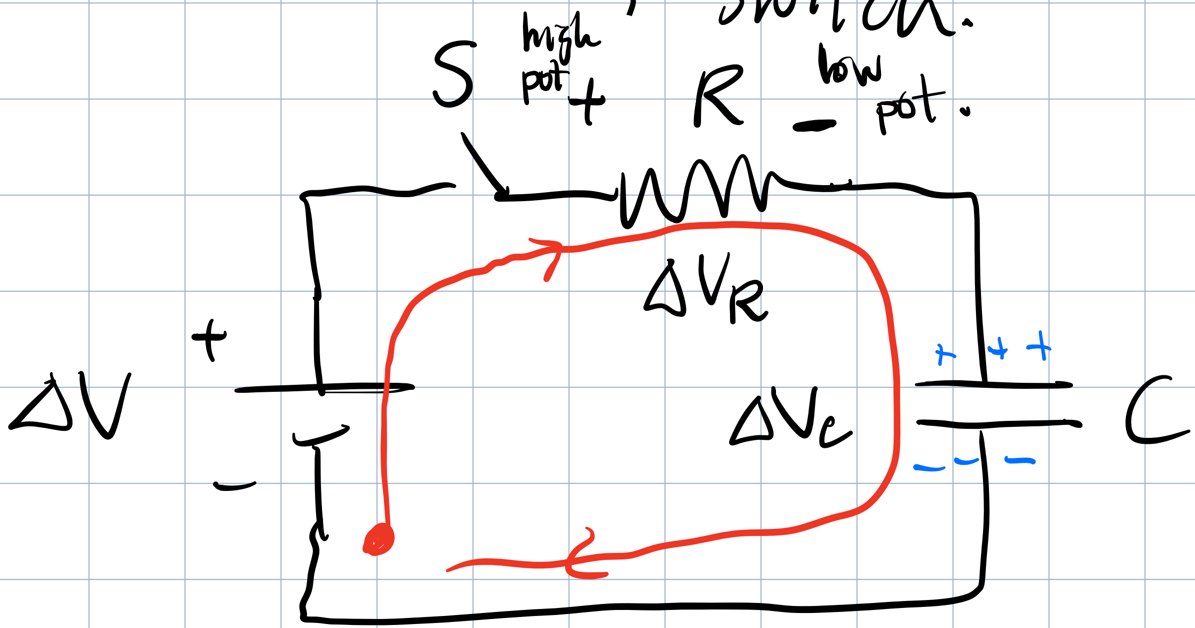
$$\Delta V = \frac{Q_{eq}}{C_{eq}}$$

$$\frac{\cancelto{1}{Q}}{C_{eq}} = \frac{\cancelto{1}{Q}}{C_1} + \frac{\cancelto{1}{Q}}{C_2} + \frac{\cancelto{1}{Q}}{C_3}$$

Series combination of capacitors

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

RC Circuit : battery, cap, resistor & switch.



If C is initially uncharged & switch S is closed at time $t=0$.

How do the current $I(t)$ & charge on the cap $Q(t)$ evolve with time?

Kirchhoff Loop Rule requires:

$$\underbrace{\Delta V}_{\text{battery}} - \underbrace{\Delta V_R}_{\text{resistor}} - \underbrace{\Delta V_C}_{\text{cap.}} = 0$$

$$\Delta V_R = IR$$

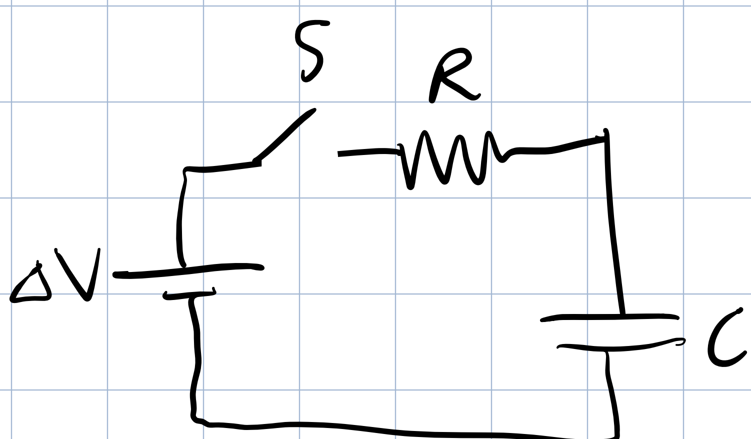
$$\Delta V_c = \frac{Q}{C}$$

$$\boxed{\Delta V - IR - \frac{Q}{C} = 0}$$

Recall $I = \frac{\Delta Q}{\Delta t}$

$$\therefore \Delta V = R\frac{\Delta Q}{\Delta t} + \frac{Q}{C}$$

This expression can be used to determine how charge changes w/ time.

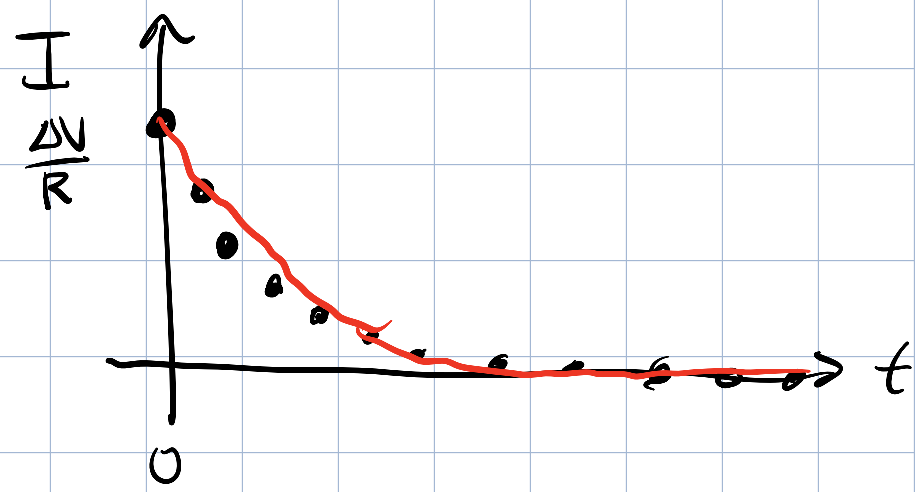


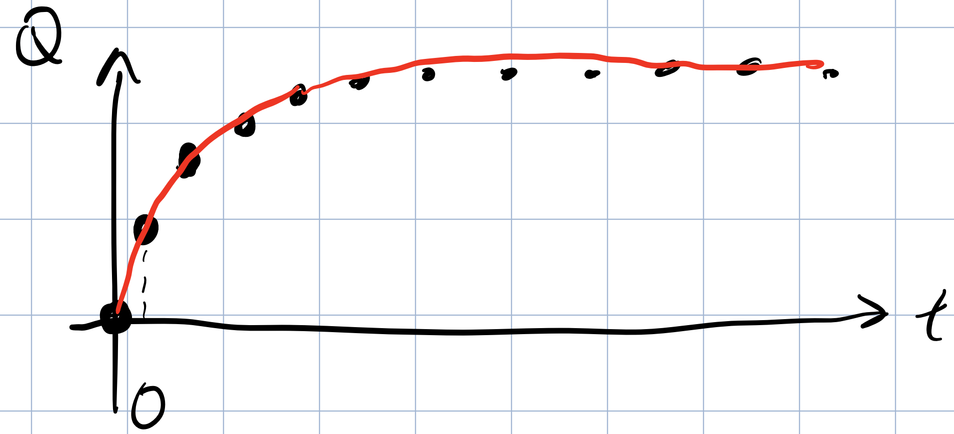
If S is just closed $\Delta V_c = 0$ (initially uncharged)

Initially, $\Delta V_R = \Delta V$ (at $t=0$)

$$\therefore IR = \Delta V$$

$$I = \frac{\Delta V}{R} \quad \text{(large current)}$$





After a short time Δt, capacitor accumulates some charge $\Delta Q = I \Delta t$

Now, voltage ΔV_C across capacitor is non-zero.
$\therefore$ Voltage across resistor & the current decrease.