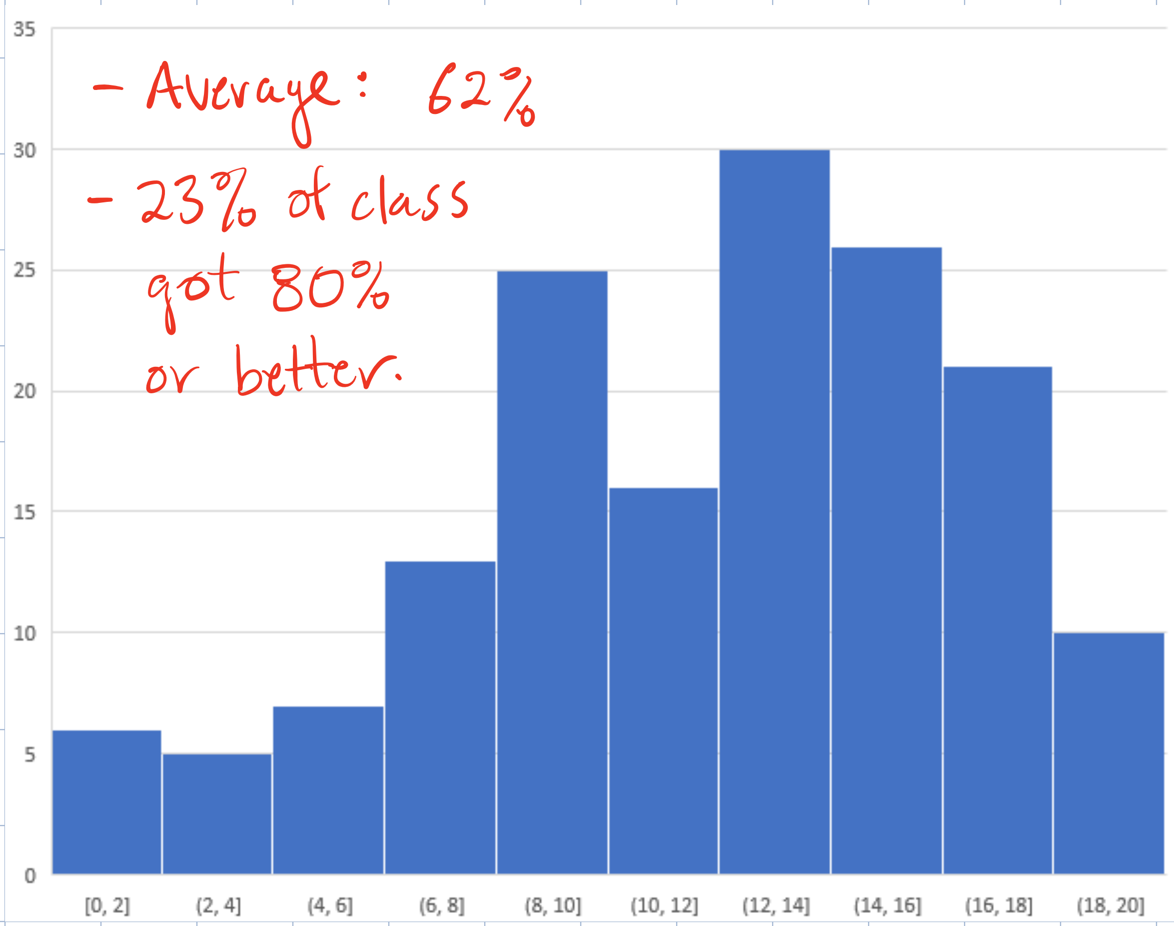
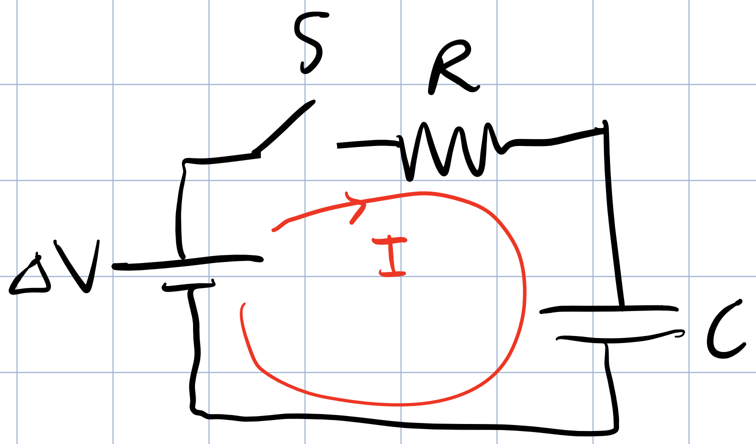
PHYS 121 March 4, 2024

- Complete PrairieLearn HW by Friday @ 23:59
- No Pre-Lab # 5
- Tutorials resume this week.

Midterm dist'n



Last Time: RC Circuit.



$$C = \frac{Q}{\Delta V_c}$$

$$\Delta V_c = \frac{Q}{C}$$

Loop Rule $\quad \Delta V - IR - \frac{Q}{C} = 0 \quad *$

Eq'n (0).

Solve Eq (0) for current I.

$$I = \underbrace{\frac{\Delta V}{R}}_{\equiv I_0} - \frac{Q}{RC}\Big\} \equiv \tau$$

(τ tau) time constant.

current at $t=0$ when the switch is closed & capacitor volt. is zero.

Define $I_0 = \frac{\Delta V}{R}$ $\quad \tau = RC$

$\therefore$ the current at any time in our RC circuit is given by:

$$\boxed{I = I_0 - \frac{Q}{\tau}} \quad (1)$$

Since $I = \frac{\Delta Q}{\Delta t}$, we can calc. how much charge is added to the cap. in time Δt using:

$$\boxed{\Delta Q = I \Delta t} \quad (2)$$

After finding ΔQ, know that new charge on the cap is the original value of Q plus ΔQ

$$Q \rightarrow Q + \Delta Q$$

Likewise $\quad t \rightarrow t + \Delta t$

Strategy:

Start w/ known values of

ΔV, R, C $\Rightarrow$ constants.

Pick a value for time interval Δt

Calc. $I_0 = \frac{\Delta V}{R}$ $\quad \tau = RC \Leftarrow$ constants

Start w/ $Q = 0$ at $t = 0$

1. Use Eq. 1 to find I.
2. Use Eq. 2 to find ΔQ (change in cap. charge during time interval Δt).
3. Update the value of $Q \Rightarrow Q + \Delta Q$
4. Update the time $t \Rightarrow t + \Delta t$
5. Return to step 1. & interate until Q reaches its max. value & $I \to 0$.

See course website for an example implementation.

Exact Solution

$$I = I_0 - \frac{Q}{\tau} \qquad (1)$$

$$I = \frac{dQ}{dt}$$

$$\frac{dQ}{dt} = I_0 - \frac{Q}{\tau}$$

$$dQ = \left(I_0 - \frac{Q}{\tau}\right) dt$$

$$\frac{dQ}{I_0 - \frac{Q}{\tau}} = dt$$

Integrate

$$\int_{Q=0}^{Q} \frac{dQ}{I_0 - \frac{Q}{\tau}} = \int_{t=0}^{t} dt$$

substitution $\quad u = I_0 - \frac{Q}{\tau} \qquad du = \frac{-1}{\tau} dQ$

$$dQ = -\tau du$$

when $Q=0$, $u = I_0$

$Q=Q$, $u = I_0 - \frac{Q}{\tau}$

$$\int_{u=I_0}^{I_0-\frac{Q}{\tau}} \frac{-\tau\, du}{u} = t$$

$$\int_{u=I_0}^{I_0-\frac{Q}{\tau}} \frac{du}{u} = -\frac{t}{\tau}$$

$$\ln u \Big|_{I_0}^{I_0-\frac{Q}{\tau}} = -\frac{t}{\tau}$$

$$\Rightarrow \ln\left(I_0 - \frac{Q}{\tau}\right) - \ln(I_0) = -\frac{t}{\tau}$$

$$\therefore \ln\left(\frac{I_0 - \frac{Q}{\tau}}{I_0}\right) = -\frac{t}{\tau}$$

$$\ln\left(1 - \frac{Q}{\tau I_0}\right) = -\frac{t}{\tau}$$

solve for Q.

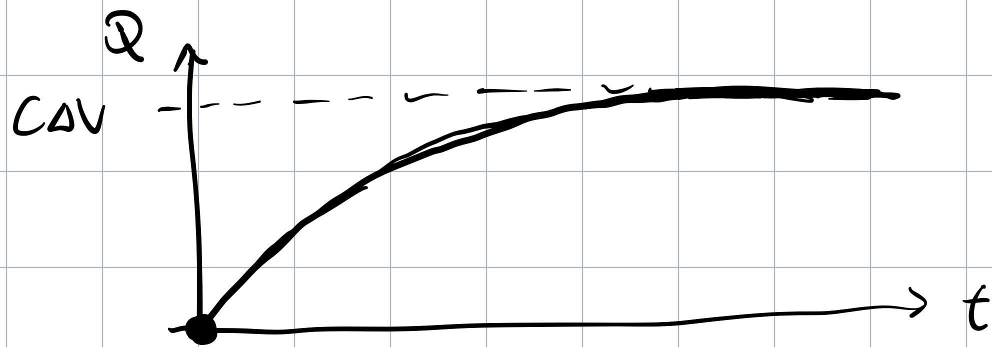
$$1 - \frac{Q}{\tau I_0} = e^{-t/\tau}$$

$$\therefore \frac{Q}{\tau I_0} = 1 - e^{-t/\tau}$$

Finally $Q = \tau I_0 \left(1 - e^{-t/\tau}\right)$

$$= (\cancel{R}C)\frac{\Delta V}{\cancel{R}}\left(1 - e^{-t/\tau}\right)$$

$$\boxed{\therefore Q = C\Delta V\left(1 - e^{-t/\tau}\right)}$$



$$I = \frac{dQ}{dt} = \frac{d}{dt}\left(C\Delta V\left(1 - e^{-t/\tau}\right)\right)$$

$$= C\Delta V\left[\cancelto{0}{\frac{d}{dt}(1)} - \underbrace{\frac{d}{dt}\left(e^{-t/\tau}\right)}_{-\frac{1}{\tau}e^{-t/\tau}}\right]$$

$$\therefore\ I = C\Delta V\,\frac{1}{\tau}e^{-t/\tau}$$

$$= \cancel{C}\Delta V\,\frac{1}{R\cancel{C}}\,e^{-t/\tau}$$

$$\boxed{I = I_0 e^{-t/\tau}}$$ current in RC circuit.