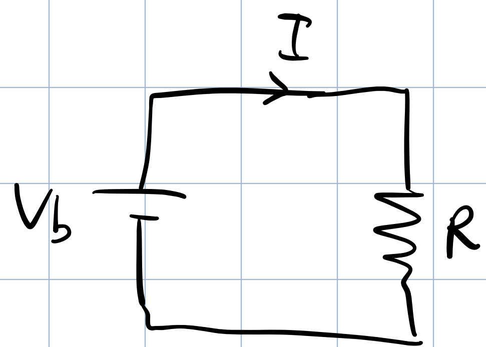
PHYS 121 March 6, 2024

- Complete PrairieLearn HW by Friday @ 23:59
- No Pre-Lab # 5

Recall :



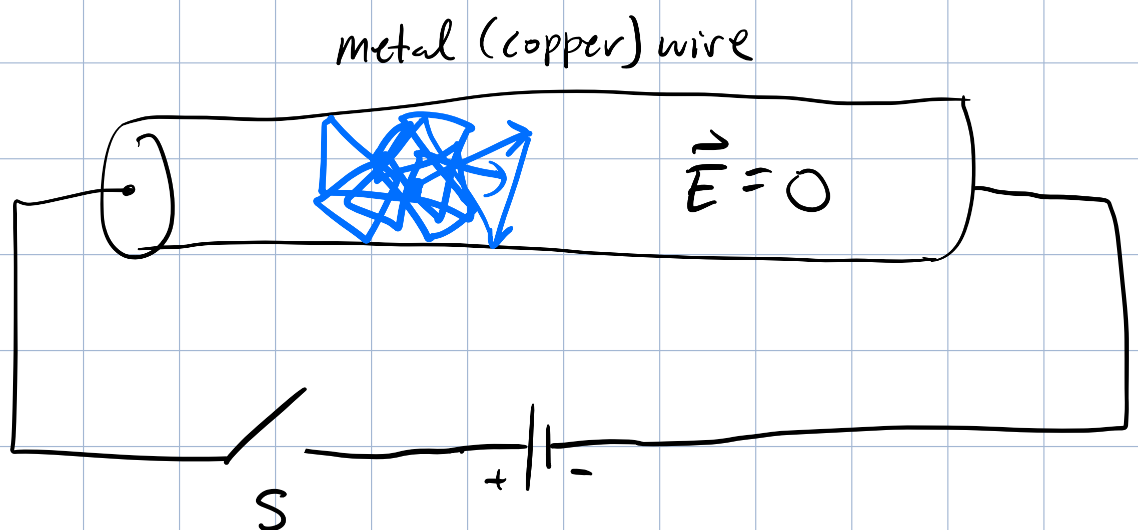
$$\boxed{V_R = IR}$$

voltage across resistor R

$$I = \frac{dQ}{dt}$$

Today: OSUPv2 Section 9.2

Model of Conduction in Metals

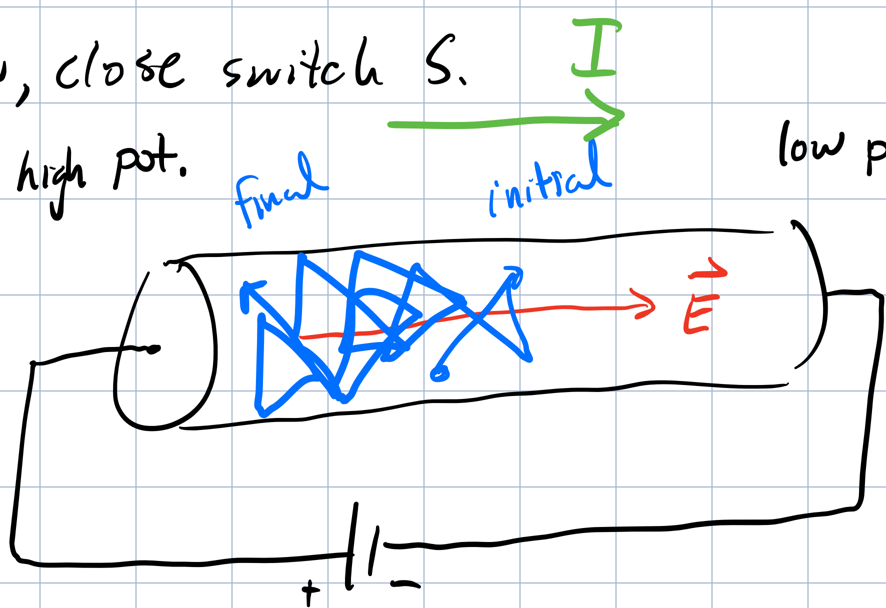


With S open, there is no voltage across wire/resistor & $\vec{E} = 0$. $\Rightarrow$ Conductor is in equil. No net flow of charge.

In a conductor, electrons constantly collide w/ atoms/impurities within the metal. After a collision, the dir'n of motion of e^- is randomized. (Brownian motion).

In this case, there is no net motion of charge in any particular dir'n $\Rightarrow I = 0$.

Now, close switch S.

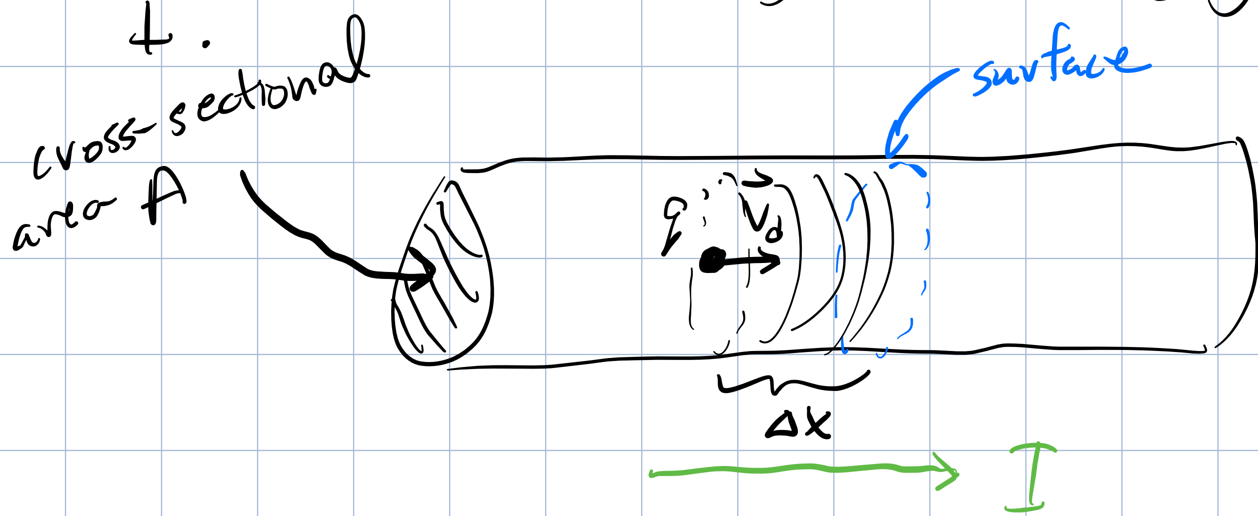


Pot. diff. across wire, establishes an $\vec{E}$ in the conductor. This field exerts force on mobile electrons causing a net flow of charge or a current.

Non-equilibrium situation.

The non-zero $\vec{E}$ tends to cause electrons to ~~be~~ drift in the dir'n opp. the $\vec{E}$ which results in a current in the dir'n of $\vec{E}$.

Consider a conducting wire carrying current I.



In time Δt, the total charge to cross surface is:

$$\Delta Q = I \Delta t \quad (1)$$

Consider a pos. charge q moving w/ "drift" velocity v_d in the dir'n of I.

In a time Δt, q moves a dist $\underline{\Delta x = v_d \Delta t}$

Any charge to the left of the blue surface, but within Δx of it, will also cross the surface in time Δt.

The shaded volume is $A \Delta x$.
If the conductor has an electron number density:

$$n = \frac{\#\text{ of electrons}}{\text{volume}}$$

then the total no. of electrons in shaded region is:

$$N_e = n \underbrace{A \Delta x}_{\text{shaded volume}}$$

no. of e^- that cross surface in time Δt.

$\frac{\text{no. } e^-}{\text{volume}}$

Since each e^- has charge e,

$$\Delta Q = eN_e = enA\underbrace{\Delta x}_{v_d \Delta t}$$

charge crossing surface in time Δt

$$\therefore \quad \Delta Q = enAv_d\,\Delta t \quad (2)$$

Eq'ns (1) & (2) calculate the same quantity and must be equal:

$$I\cancel{\Delta t} = env_dA\cancel{\Delta t}$$

$$\boxed{I = env_dA} \iff v_d = \frac{I}{enA}$$

Eg. For a copper wire of diameter $d = 1\,mm$ & $I = 1\,A$, what is v_d?

For copper, the electron number density is

$$n = 8.3 \times 10^{28} \frac{e^-}{m^3}$$

$$e = 1.6 \times 10^{-19}\ C$$

$$V_d = \frac{I}{enA} = \frac{I}{en\left(\pi\left(\frac{d}{2}\right)^2\right)} = 9.6 \times 10^{-5}\ m/s$$
$$\approx 0.1\ mm/s$$

Return to $I = e n V_d A$.

Note that I depends on the geometry of the wire (A). Sometimes convenient to define a "current density" $J = \frac{I}{A} = e n V_d$

Note that current density is a vector w/ dir'n in the same dir'n as drift velocity.

$$\vec{J} = e n \vec{V_d}$$

For some materials (metals), the current density J is prop. to the electric field E in the wire:

$$J \propto E$$

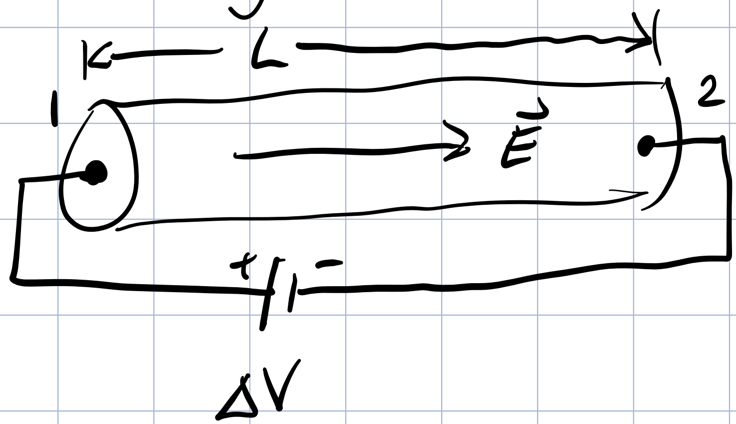
The constant of proportionality is called the conductivity & is given the symbol σ (sigma).

$$J = \sigma E \qquad (3)$$

Materials for which this proportionality holds are called "ohmic" materials & they obey "Ohm's Law".

Know $J = \dfrac{I}{A}$ (3a)

Consider the following conductor



$$\Delta V = -\int_1^2 \vec{E} \cdot d\vec{s}$$

If we assume that $\vec{E}$ in conductor is constant, then

$$|\Delta V| = E L$$

$$E = \frac{\Delta V}{L} \quad (3b)$$

Sub (3a) & (3b) into (3)

$$\frac{I}{A} = \sigma \frac{\Delta V}{L}$$

Solve for ΔV, voltage across the wire:

$$\Delta V = I\underbrace{\left(\frac{1}{\sigma}\frac{L}{A}\right)}$$

$$= I\ R$$

R the resistance of our wire of length L & cross-sec. area A.

$$\boxed{R = \frac{1}{\sigma}\frac{L}{A}}$$

$$\boxed{\Delta V = IR}$$

Ohm's Law.

$$[R] = \Omega = \frac{V}{A}$$

$$[\sigma] = \frac{1}{[R]}\frac{[L]}{[A]} = \frac{\cancel{m}}{\Omega\ m^{\cancel{2}}} = \frac{1}{\Omega \cdot m}$$

conductivity.

Often resistance R is expressed in terms of resistivity $\rho = \frac{1}{\sigma}$

$$\therefore \boxed{R = \rho \frac{L}{A}} \qquad [\rho] = \Omega \cdot m$$