PHYS 121 March 11, 2024

- Complete PrairieLearn HW by Friday @ 23:59
- Complete Pre-Lab #6 before the start of Lab #6
- Quiz #2 will be on Wednesday, March 20
  ⇒ See course website for details.

Recall Kirchhoff Rules:

1. Loop Rule - sum of voltage changes around a closed loop is zero

$$\sum_i \Delta V_i = 0$$

2. Junction Rule - total current into a junction equals total current leaving the junction

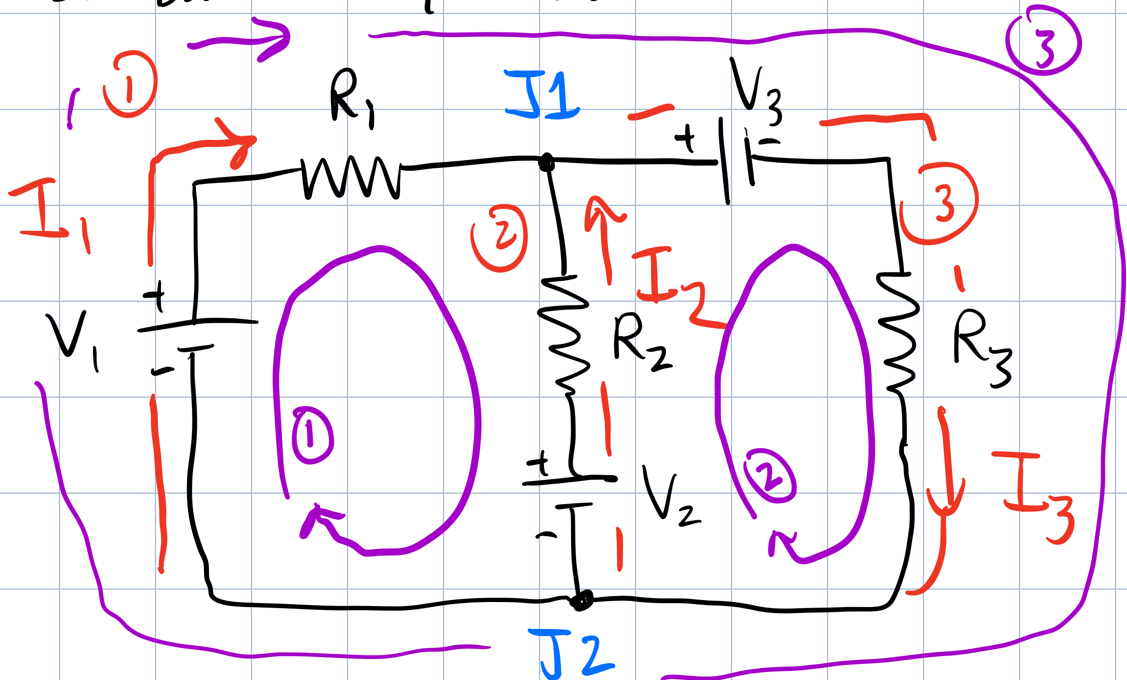
$$I_{tot,\,in} = I_{tot,\,out}$$

Last Time:

1. If cross resistor in dir'n of current $\Rightarrow \Delta V_R = -IR$.

2. If cross resistor in dir'n opposite of current $\Rightarrow \Delta V_R = +IR$.

Eg. Consider the following circuit consisting of batteries & Resistors.



Typically in a circuit like this will have some unknowns to solve for (currents, battery voltage, resistor values)

Step a system of N eq'ns to solve for N unknowns.

Eg. In the circuit above find the current in each branch of circuit (current in each resistor).

Step 1: Assign a current to each branch of circuit. Label currents & pick a dir'n.

If at the end of the problem we calc. a neg. value for a current, it just means the we chose the wrong dir'n. All other calc. quantities will be correct.

Step 2: Identify the unknowns.

In this example, assume battery voltages and resistor values are known.

Unknowns are: I_1, I_2, I_3.

To solve for 3 unknowns, need to construct a system of 3 indep. equations involving the 3 unknowns.

Step 3: Apply the junction to start building our system of eq'ns.

Eq. J1: $I_{in} = I_{out}$

$$\boxed{\underbrace{I_1 + I_2}_{I_{in}} = \underbrace{I_3}_{I_{out}}} \quad \text{a}$$

J2: $\underbrace{I_3}_{I_{in}} = \underbrace{I_1 + I_2}_{I_{out}}$

Same result from J1 (no new information).

Step 4: Use the loop rule to find the other two required eq'ns.

Loop ①: $$0 = \underbrace{+V_1}_{\text{neg-to-pos terminal}} - \underbrace{I_1 R_1}_{\text{in the dir'n of } I_1} + \underbrace{I_2 R_2}_{\text{opp. dir'n of } I_2} - \overbrace{V_2}^{\text{pos. to neg. term}}$$

Loop ②: $$0 = +V_2 - I_2 R_2 - V_3 - I_3 R_3$$

Loop ③: $$0 = +V_1 - I_1 R_1 - V_3 - I_3 R_3$$

3 loop eq'ns. Claim that there are only 2 independent eq'ns.

Consider (Loop ①) + (Loop ②)

$$\Rightarrow \left(+V_1 - I_1 R_1 + \cancel{I_2 R_2} - \cancel{V_2}\right)$$

$$+ \left(\cancel{V_2} - \cancel{I_2 R_2} - V_3 - I_3 R_3\right) = 0 + 0$$

$$\therefore \quad V_1 - I_1 R_1 - V_3 - I_3 R_3 = 0$$

(Loop 3) eq'n.

Since we can construct the third eq'n using the first two, only have two indep. eq'ns.

Step 5: Finalize system of 3 eq'ns.

→ Use one junction rule

→ Two loop rule

For the current example:

$$I_1 + I_2 = I_3 \qquad (a)$$

$$V_1 - I_1 R_1 + I_2 R_2 - V_2 = 0 \quad (b)$$

$$V_2 - I_2 R_2 - V_3 - I_3 R_3 = 0 \quad (c)$$

Step 6: Solve the system of 3 eq'ns for the 3 unknowns (just a math problem).

Eg. Take the circuit drawn above, assume:

$$V_1 = V \qquad R_1 = R$$
$$V_2 = 2V \qquad R_2 = 2R$$
$$V_3 = 3V \qquad R_3 = R$$

System of 3 eq'ns.

$$I_1 + I_2 = I_3 \quad (a)$$

$$V - I_1 R + 2 I_2 R - 2V = 0$$

$$-\frac{V}{R} - I_1 + 2 I_2 = 0 \quad (b)$$

$$2V - 2I_2 R - 3V - I_3 R = 0$$

$$-\frac{V}{R} - 2I_2 - I_3 = 0 \quad \text{(c)}$$

To solve this system, start by using two of the eq'ns to eliminate one of the unknowns.

Sub (a) in (c) to eliminate I_3

$$-\frac{V}{R} - 2I_2 - (I_1 + I_2) = 0$$

$$\therefore -\frac{V}{R} - I_1 - 3I_2 = 0 \quad \text{(d)}$$

Now, eq'ns (b) & (d) form a system of two eq'ns & two unknowns.

Take (b) − (d) to eliminate I_1.

$$\left(\cancel{-\frac{V}{R}} \cancel{-I_1} + 2I_2\right) - \left(\cancel{-\frac{V}{R}} \cancel{-I_1} - 3I_2\right) = 0$$

(b) (d)

$$\therefore\ 2I_2 + 3I_2 = 0$$

$$5I_2 = 0$$

$$\Rightarrow \boxed{I_2 = 0}$$

Sub known value of I_2 into (b) or (d) to find I_1.

$$\text{(b)}\quad -\frac{V}{R} - I_1 + 2(\overset{I_2}{\downarrow}\!\!\!\!0) = 0$$

$$\boxed{\therefore\ I_1 = -\frac{V}{R}}$$

same!

(d) $-\frac{V}{R} - I_1 - 3(\underset{I_2}{0}) = 0$

$\therefore I_1 = -\frac{V}{R}$

Sub known values of I_1 & I_2 into (a)

$I_1 + I_2 = I_3$

$-\frac{V}{R} + 0 = I_3 \Rightarrow \boxed{I_3 = -\frac{V}{R}}$

b/c I_1, I_3 are neg., these currents run in the opp. dir'n chose at start of the problem.