

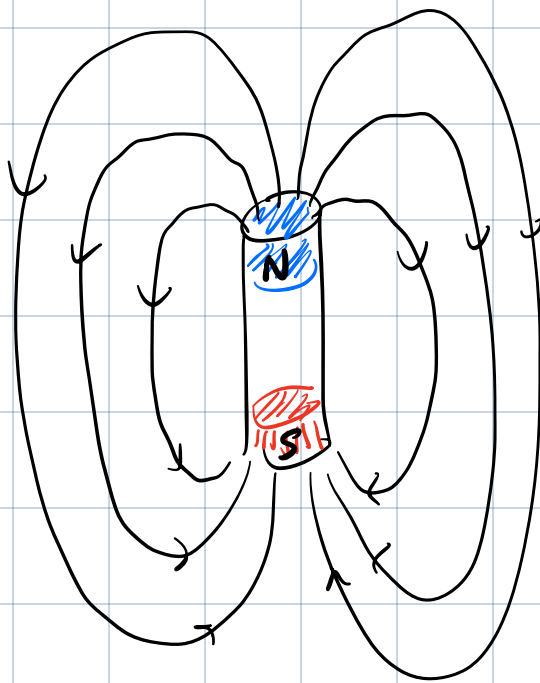
PHYS 121

March 15, 2024

- ✓ - Complete PrairieLearn HW by 23:59 today
- ✓ - Complete Pre-Lab #7 before the start of Lab #7
- ✓ - Quiz #2 will be on Wednesday, March 20  
⇒ See course website for details.

Last Time:

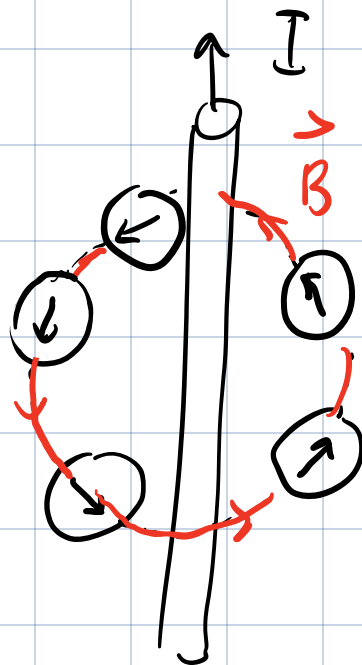
Magnets always come in pairs of N & S poles → no isolated poles or monopoles



Magnetic field lines  $\vec{B}$  exit North poles & enter South poles.

- Current or moving charges are a source of magnetic fields. Static or stationary charges do not create magnetic fields.

Continue with thinking about a compass next to a wire with a current  $I$ .



The current creates a magnetic field  $\vec{B}$  that loops around the wire.

This magnetic field causes the compass needle to align with the field due to the current.

If we reverse the dir'n of  $I$ , the magnetic field changes dir'n & compass needles would

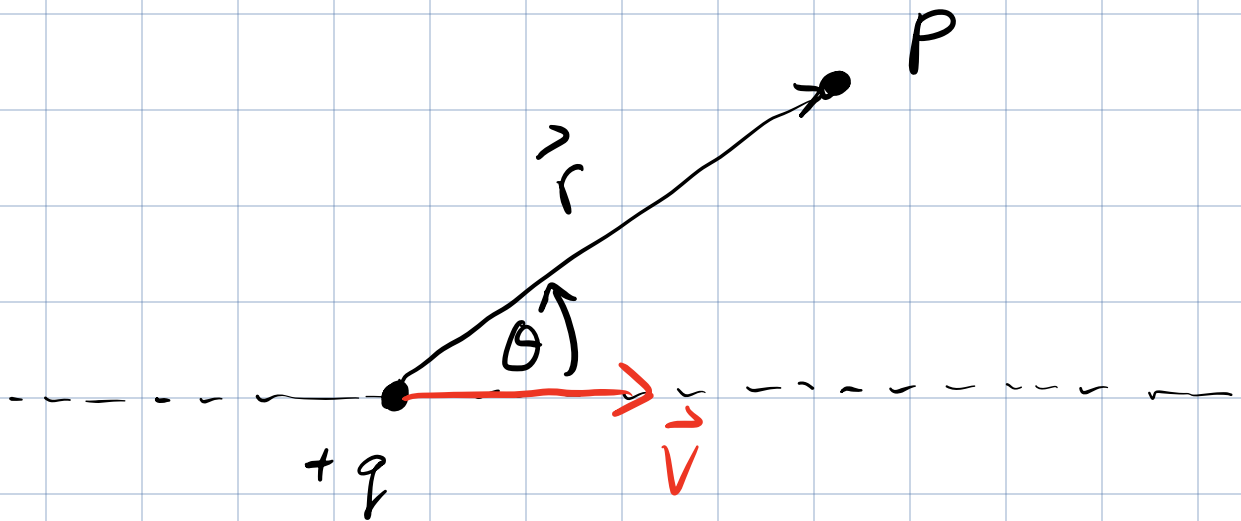
rotate by  $180^\circ$ .

To find the correct dir'n of  $\vec{B}$  due to a current, we use the right hand rule (RHR)

1. Pt. thumb of right hand in the dir'n of  $\vec{I}$ .
2. Fingers naturally curl in dir'n of resulting  $\vec{B}$ -field.



Imagine a pt. charge  $q$  moving in a straight line with velocity  $\vec{v}$ .



What is  $\vec{B}$  at  $P$  due to moving charge?

Find that the strength of magnetic field  $\vec{B}$  at  $P$  is proportional to:

- the charge  $q$
- the speed  $|\vec{v}|$  of the charge.
- $\frac{1}{r^2}$
- $\sin \theta$  (perhaps a little surprising)

$$\theta = 90^\circ$$

$$\sin \theta = 1 \text{ (max)}$$

B is max

$$B \propto \frac{qV}{r^2} \sin \theta$$

$$0 < \theta < 90^\circ$$

$$0 < \sin \theta < 1$$

$$0 < B < B_{\max}$$

$$90 < \theta < 180$$

$$0 < \sin \theta < 1$$

$$0 < B < B_{\max}$$

$$B = 0$$

$$\text{b/c } \theta = 0$$

$$\left\{ \begin{array}{l} \sin \theta = 0 \end{array} \right.$$

Through careful measurements, we can find the constant of proportionality:

$$B = \frac{\mu_0}{4\pi} \frac{qV \sin \theta}{r^2}$$

$\frac{\mu_0}{4\pi}$   
constant  
of proportionality

$\mu_0$  is called the "permeability of free space".

$$\mu_0 = 4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}$$

$$\frac{\mu_0}{4\pi} = 1 \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}$$

$$[B] = [\mu_0] \left[ \frac{q v}{r^2} \right]$$

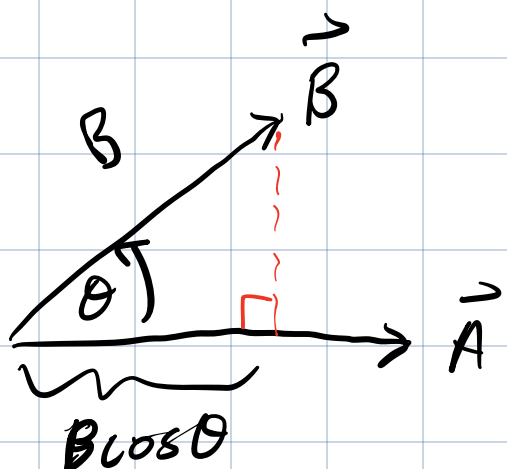
$$= \frac{\text{T} \cdot \text{m}}{\text{A}} \left[ \frac{\text{C} \cdot \text{m}}{\text{s}} \frac{1}{\text{m}^2} \right] = \text{T}$$

Magnetic fields are measured in units of Tesla (T).

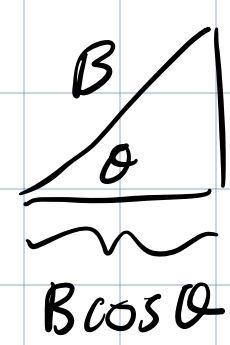
# Cross-Product

The cross-product is a product between two vectors.

First, recall the dot product between two vectors



$B \cos \theta$



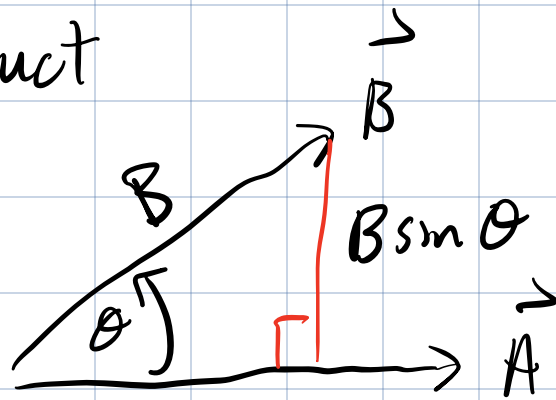
$B \cos \theta$

$$\begin{aligned}\vec{A} \cdot \vec{B} &= |\vec{A}| |\vec{B}| \cos \theta \\ &= AB \cos \theta\end{aligned}$$

Note that result of  $\vec{A} \cdot \vec{B}$  is a scalar (a number).

The dot product selects the component of  $\vec{B}$  that is parallel to  $\vec{A}$

# Cross Product



$$|\vec{A} \times \vec{B}| = AB \sin \theta$$

← just the magnitude of the cross product.

not ordinary multiplication

The cross product selects the component of  $\vec{B}$  that is  $\perp$  to  $\vec{A}$ .

One important difference between dot & cross products is that the result of the cross product is another vector.

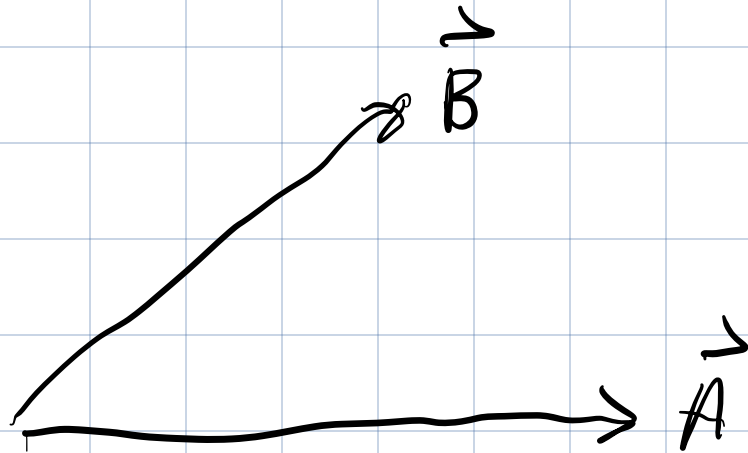
In general

$$\vec{A} \times \vec{B} = \vec{C}$$



$$|\vec{C}| = AB \sin \theta$$

Vectors  $\vec{A}$  &  $\vec{B}$  lie in a plane.



In this  
the plane  
is the  
screen.

$\vec{C}$  will be  $\perp$  to this plane or  
(equivalently)  $\vec{C}$  is  $\perp$  to both  $\vec{A}$   
&  $\vec{B}$ .

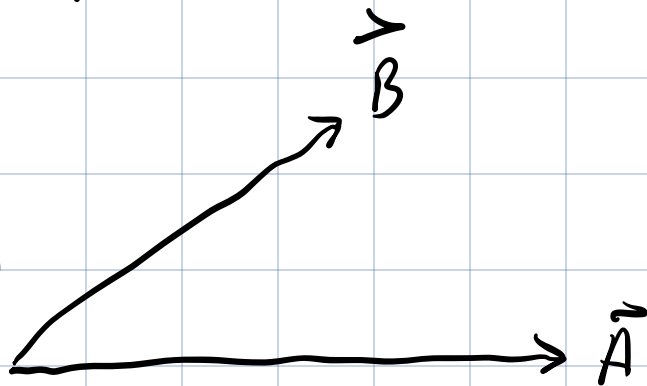
To determine whether  $\vec{C}$  is into or  
out of the screen, we use a second  
RHR.

Place right hand/arm in dir'n of  $\vec{A}$  (the first vector in the cross product)

Curl fingers of right hand so that they are parallel to  $\vec{B}$  (second vector in the cross product)

Thumb points in the dir'n of  $\vec{C}$ .

$\vec{C} = \vec{A} \times \vec{B}$  points out of the screen.



$\vec{D} = \vec{B} \times \vec{A}$  points into the screen.