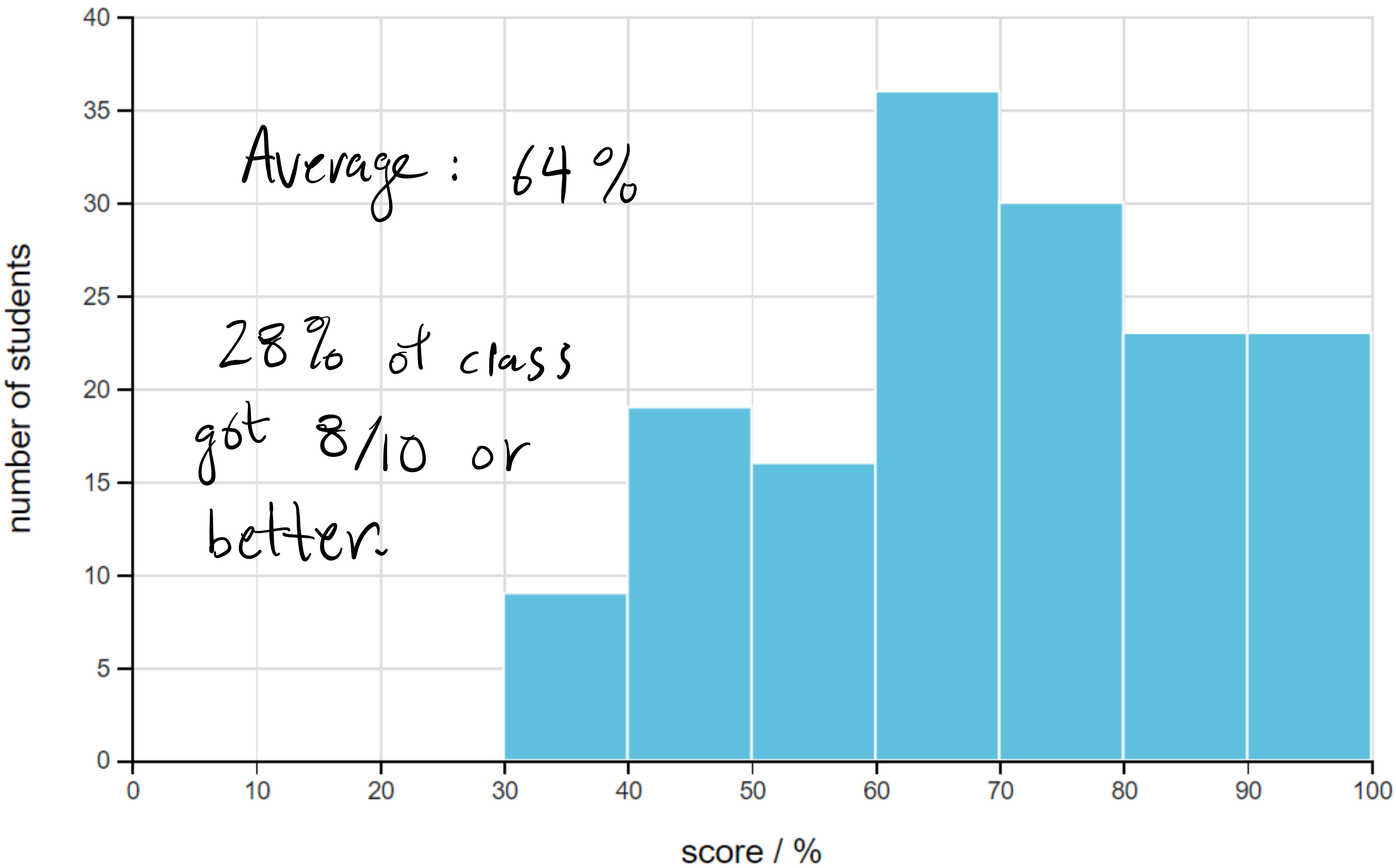
PHYS 121

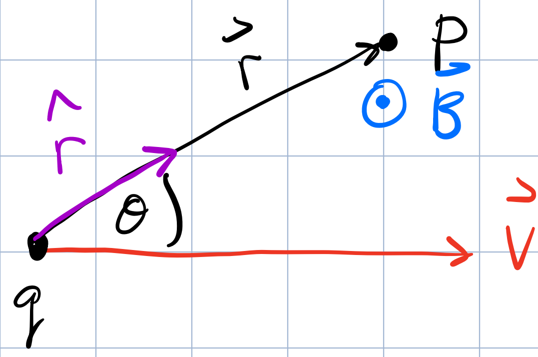
March 22, 2024

- The next PrairieLearn HW is due Fri., Mar. 29
- Complete Pre-Lab #8 before the start of Lab #8
- If completing the Hands-On bonus project, send me the link to your YouTube video by Monday, Apr. 8 @ 23:59.



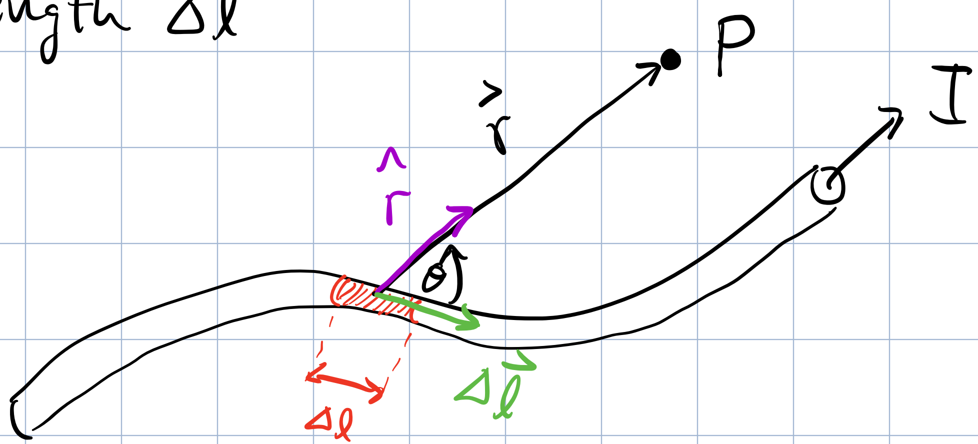
Last Time:

$\vec{B}$ @ P due to moving charge



$$\boxed{\vec{B} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \hat{r}}{r^2}}$$

$\vec{B}$ @ P due to current segment of length Δl



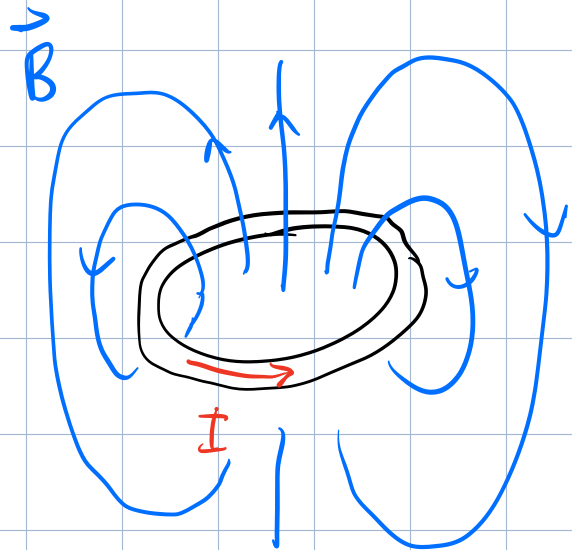
$$\boxed{\vec{B}_i = \frac{\mu_0}{4\pi} \frac{I \Delta \vec{l} \times \hat{r}}{r^2}}$$

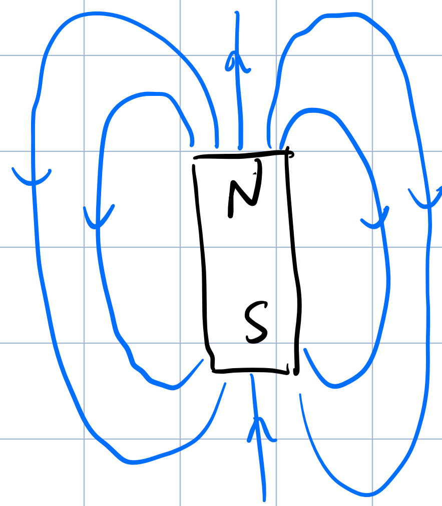
Biot-Savart Law

To get net magnetic field @ P, sum contributions from all current segments that make up the wire:

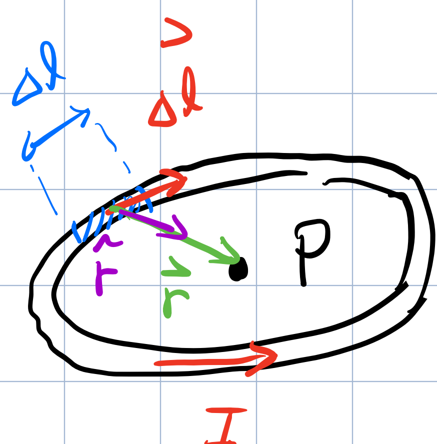
$$\vec{B}_{net} = \sum_i \vec{B}_i$$

Current Loop creates a magnetic field that is similar to that of a bar magnet





Today: Start by using Biot-Savart Law to calc. $|\vec{B}|$ at centre of a current loop.

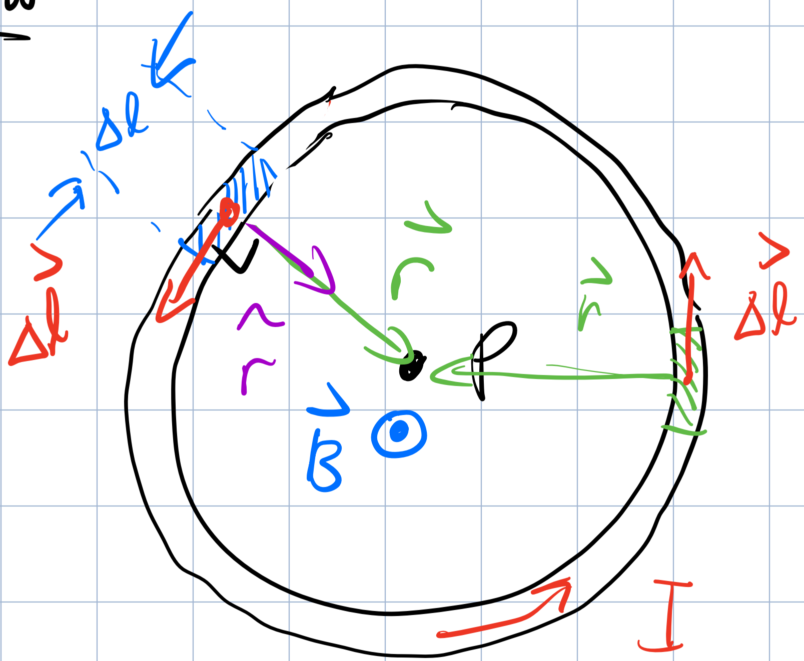


P is @ centre of loop. Find $\vec{B}$ at centre.

From Biot-Savart law $\vec{B}$ @ P due to blue section of current loop is given by:

$$\vec{B}_i = \frac{\mu_0}{4\pi} \frac{I\, \Delta\vec{l} \times \hat{r}}{r^2}$$

Top View:



First consider $\Delta\vec{l} \times \hat{r}$

Magnitude $|\Delta\vec{l} \times \hat{r}| = \underbrace{|\Delta\vec{l}|}_{\Delta l} \underbrace{|\hat{r}|}_{1} \underbrace{\sin 90°}_{1}$

$= \Delta l$

Dir'n of $\Delta\vec{l} \times \hat{r}$: By RHR $\Delta\vec{l} \times \hat{r}$ is out of the screen

$$\vec{B}_i = \frac{\mu_0}{4\pi} \frac{I \Delta l}{r^2}$$ out of screen.

due only to blue current segment.

$\vec{B}$ due to all segments needed to form the complete current loop is:

$$\vec{B} = \sum_i \vec{B}_i$$

<u>Note</u>: All current segements in the loop contribute magnetics that are out of the screen.

$$B = \sum_i B_i = \sum_i \frac{\mu_0}{4\pi} \frac{I \Delta l}{r_i^2}$$

since all segments the the same dist. r from P.

$$B = \sum_i \frac{\mu_0}{4\pi} \frac{I \Delta l}{r^2}$$

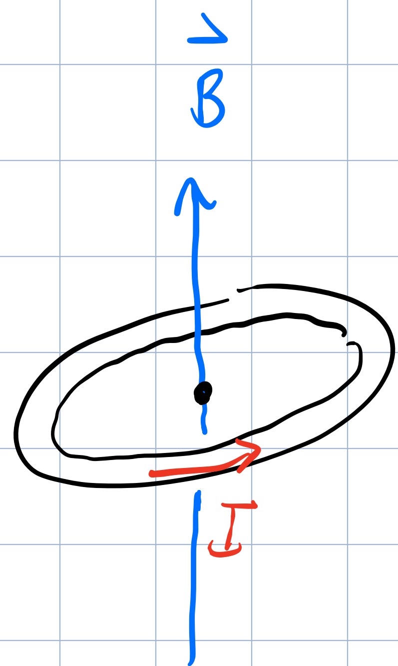
limit that $\Delta l \to 0$

$$B = \int_{loop} \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2} = \frac{\mu_0}{4\pi} \frac{I}{r^2} \underbrace{\int_{loop} dl}_{2\pi r}$$

Magnetic field at centre of a current loop is given by:

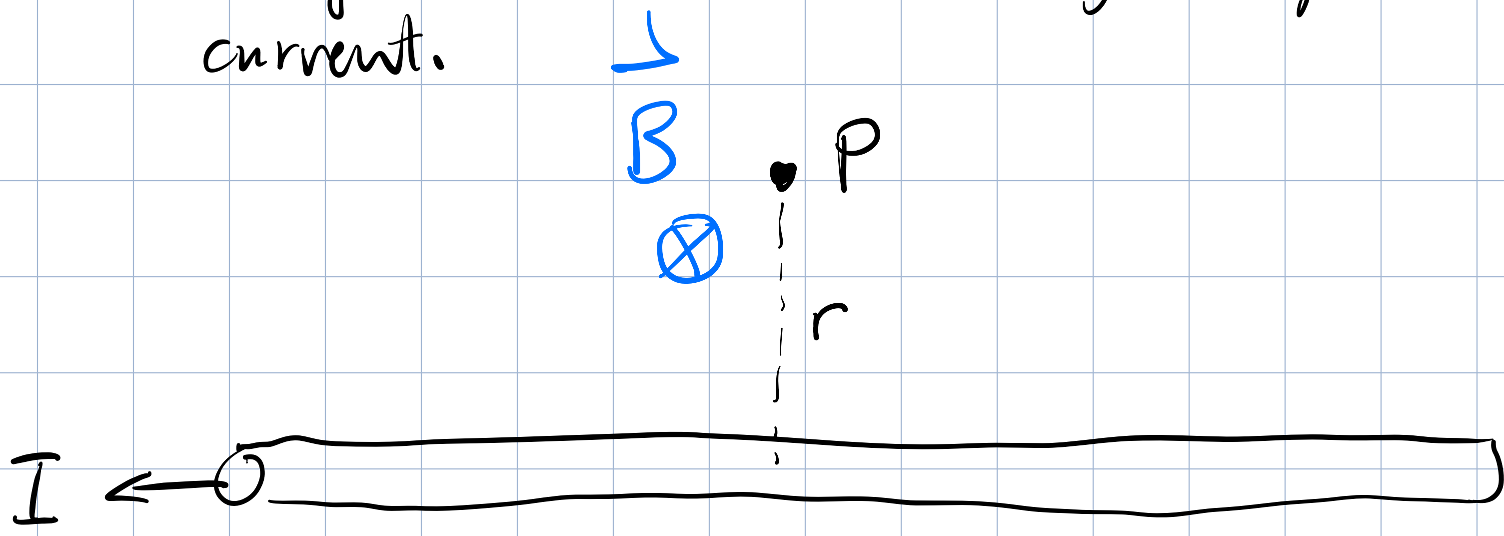
$$B = \frac{\mu_0}{4\cancel{\pi}} \frac{I}{r^{\cancel{2}}} 2\cancel{\pi}\cancel{r}$$

$$\boxed{B_{loop} = \frac{\mu_0 I}{2r}}$$



Valid only at centre of loop.

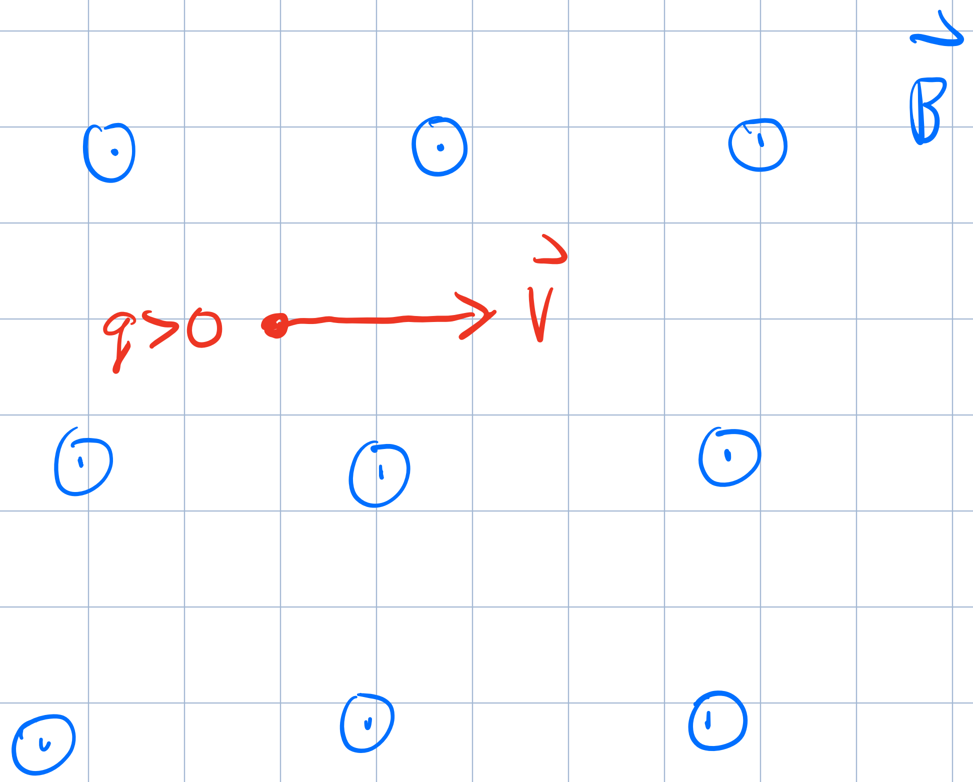
Let's also state the result for the magnetic field due to a long, straight current.



For a detailed analysis using Biot-Savart law, see OSUPv2 section 12.2.

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi r} \text{ into screen}}$$

Force on moving charge due to a magnetic field $\vec{B}$



Observations:

① Only moving charges experience a force in a magnetic field.

$$F \propto v$$

② Force changes in proportion to the value of q.

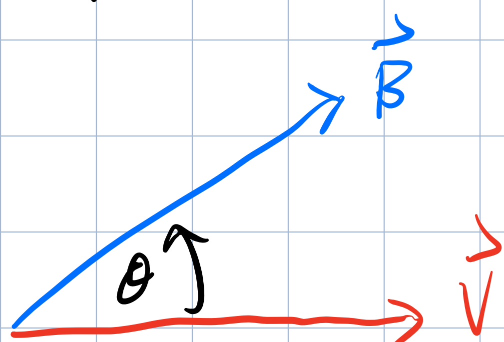
$$F \propto q$$

③ Force is proportional to $|\vec{B}|$

$$F \propto B$$

④ The value of the force depends on the angle between $\vec{v}$ & $\vec{B}$

$$F \propto \sin\theta$$



F is a maximum when $\vec{v} \perp \vec{B}$ $(\theta = 90°)$

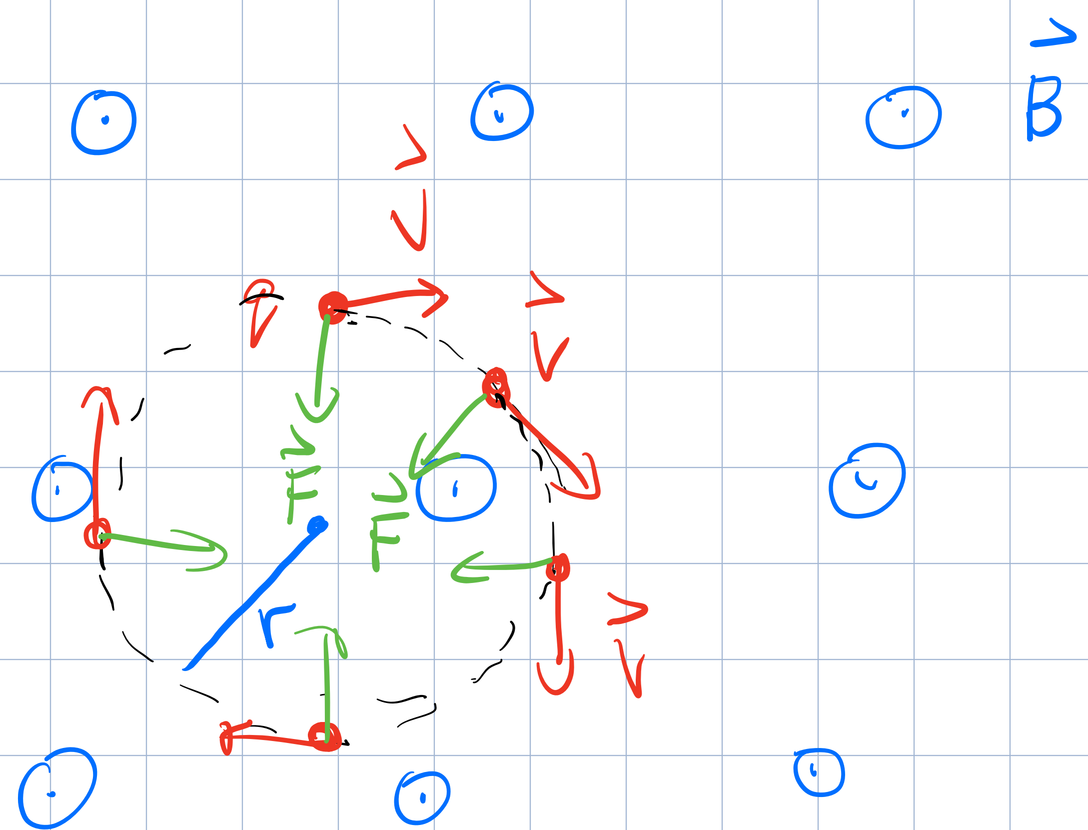
$F = 0$ when $\vec{v} \parallel \vec{B}$ $(\theta = 0°)$

$$|\vec{F}| = q\underbrace{vB\sin\theta}_{\vec{v}\times\vec{B}}$$

$$\boxed{\vec{F} = q\vec{v}\times\vec{B}}$$

Force acting on a charge moving through magnetic field $\vec{B}$.

Simplest example is a charge moving $\perp$ to a uniform $\vec{B}$



$$\vec{F} = q\vec{v} \times \vec{B}$$

Charges moving through a uniform magnetic field undergo circular motion.

$$F = |\vec{F}| = qvB \underbrace{\sin\theta}_{1} \quad 90^\circ$$

$$F = qvB = ma_c = m\left(\frac{v^2}{r}\right)$$

centripetal accleration.

$$q\cancel{v}B = \frac{m v^{\cancel{2}}}{r}$$

$$\boxed{r = \frac{mv}{qB}}$$

radius of circular motion.

The time for the charge to complete one revolution is :

period $$T = \frac{2\pi r}{v} = \frac{2\pi}{\cancel{v}}\left(\frac{m\cancel{v}}{qB}\right)$$

$$\boxed{T = \frac{2\pi m}{qB}}$$

Period of circular motion

→ indep. of v & r.