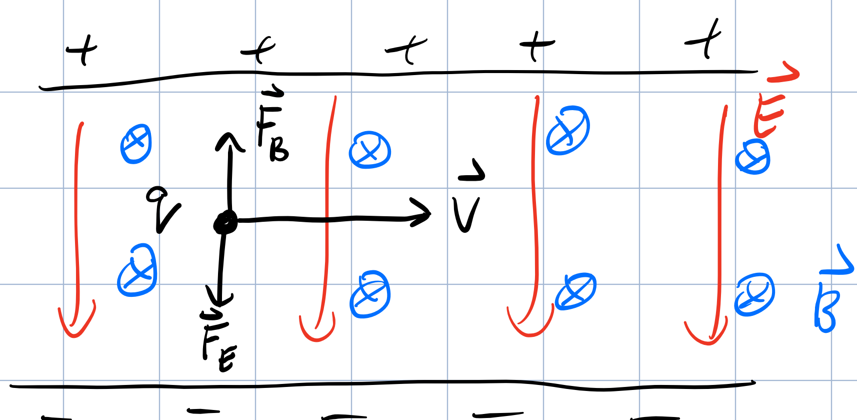
PHYS 121 April 3, 2024

- ✓ The last PrairieLearn HW is due Thur, Apr. 11 @ 23:59
- ✓ If completing the Hands-On bonus project, send me the link to your YouTube video by Monday, Apr. 8 @ 23:59.
- ✓ No tutorials this week. Last Tutorial is next week.
- ✓ PHYS 121 Labs are done!
- ✓ Complete end-of-term survey by 23:59 on Wed. Apr. 10 to receive 0.5 towards final grade.
  Link to Survey is on PHYS 121 Canvas home-page.

Last time: Velocity selector

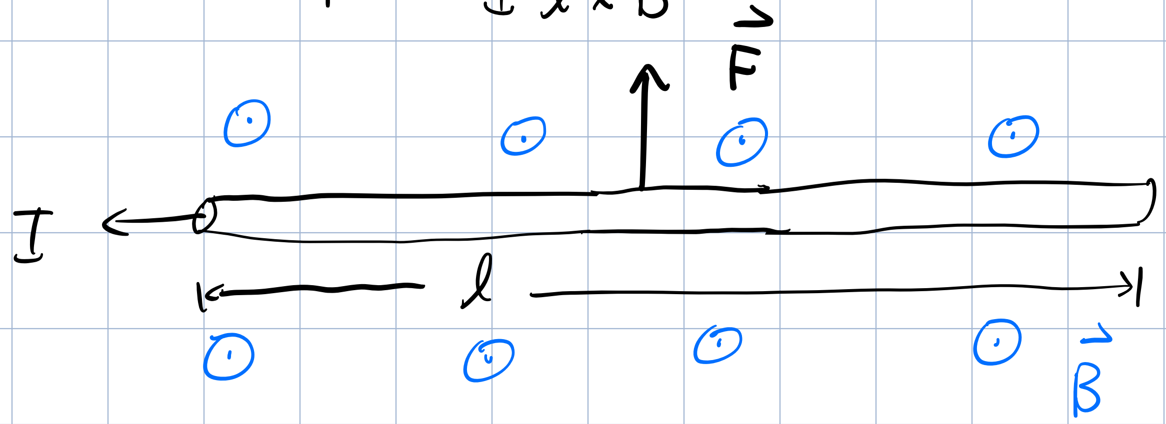


Charge passes through undeflected if

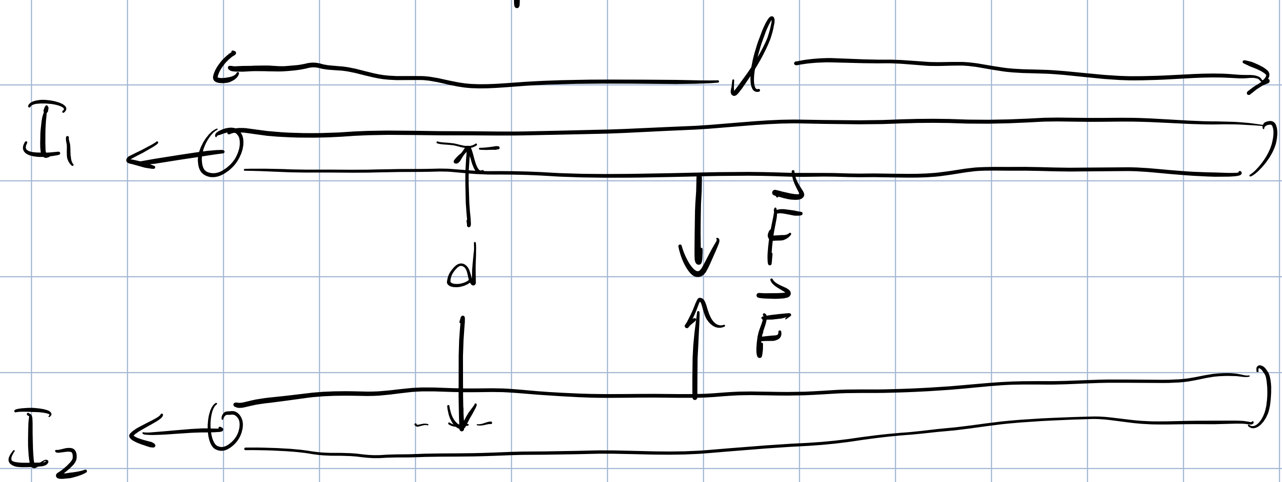
$$qE = qvB \Rightarrow v = \frac{E}{B}$$

Force on a current in a magnetic field

$$\vec{F} = I\,\vec{l} \times \vec{B}$$



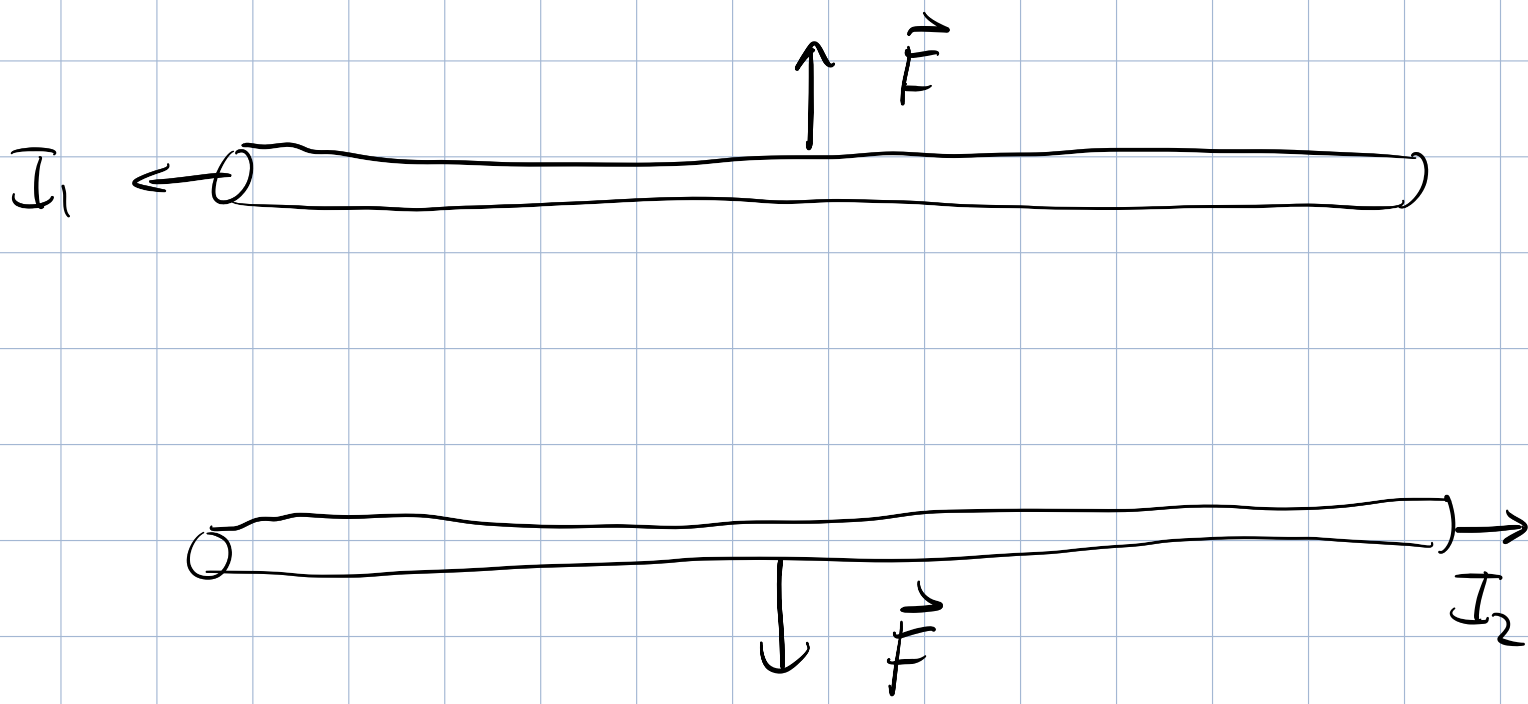
Force between parallel currents.



Parallel currents attract w/ force

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

Force between anti-parallel currents



antiparallel currents repel.

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

Today: Ampere's Law

- → Used to find $\vec{B}$ due to various configurations of currents.
- → Analogue of Gauss's Law, but for magnetic fields.

We will find that:

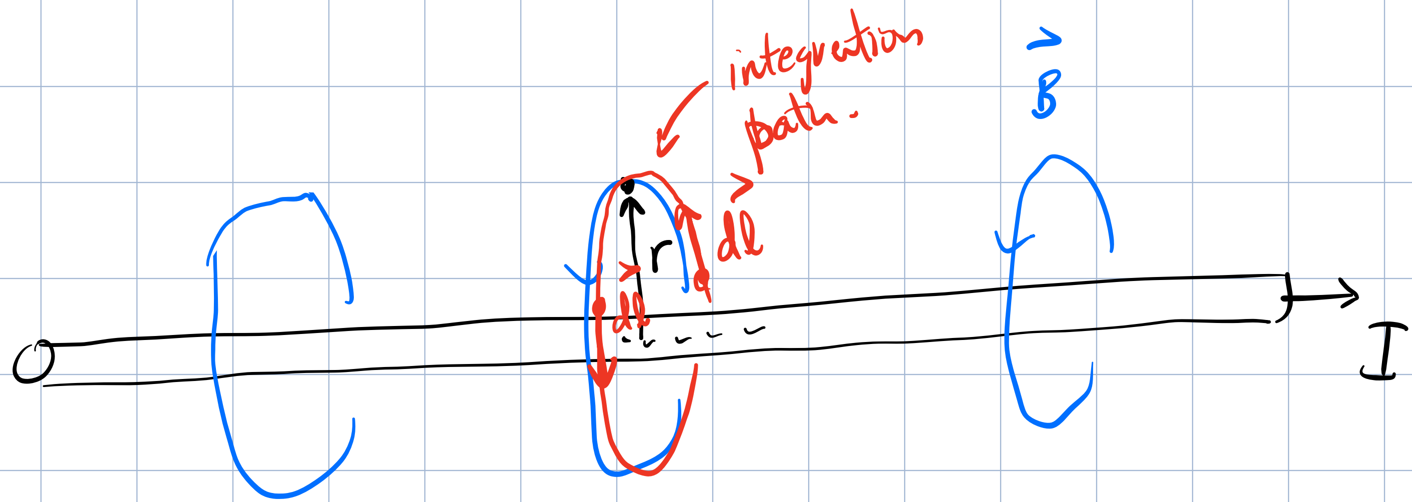
$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}}$$

Ampere's Law.

integrate around a closed loop.

Recall that $\vec{B}$ due to a long, straight current is given by

$$B = \frac{\mu_0 I}{2\pi r}$$



Try to evaluate the integral $\oint \vec{B} \cdot d\vec{l}$ for a loop that surrounds current I.

Choose loop (integration path) such that it is always $\parallel$ to $\vec{B}$.

In this case $\vec{B} \cdot d\vec{l} = B\,dl \underbrace{\cos(0)}_{1} = B\,dl$

Notice that distance from wire/current to integration path is always the same (r)

$\therefore$ $\vec{B}$ has a constant magnitude along red integration path.

$$\therefore \oint \vec{B}\cdot d\vec{l} = \oint \overset{\text{const.}}{B}\, dl = B \underbrace{\oint dl}_{2\pi r}$$

$2\pi r$ (circumference of circular path.

$$\therefore \oint \vec{B}\cdot d\vec{l} = \overset{\frac{\mu_0 I}{2\pi r}\text{ for long straight current.}}{B}(2\pi r)$$

$$= \frac{\mu_0 I}{\cancel{2\pi r}}(\cancel{2\pi r})$$

$$\therefore \oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

current that passes through the integration path / Amperian loop.

Usually the current is labelled as I_{encl}

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}}$$

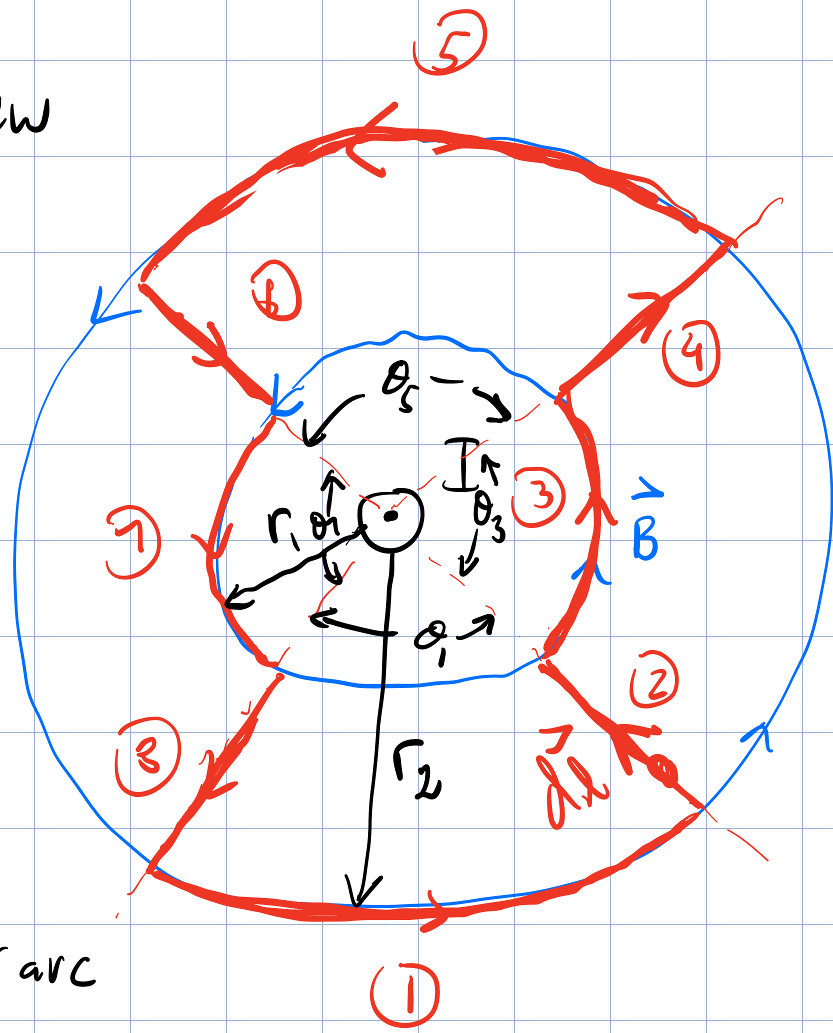
Ampere's Law

What about an integration of arbitrary shape? Does Ampere's Law still hold?

End view

$$B = \frac{\mu_0 I}{2\pi r}$$

Can form any path around I using small radial & circular arc segments.



Evaluate $\oint \vec{B}\cdot d\vec{l} = \int_{①} \vec{B}\cdot d\vec{l} + \cancel{\int_{②} \vec{B}\cdot d\vec{l}}^{\,0} + \ldots$

$$+ \int_{⑦} \vec{B}\cdot d\vec{l} + \cancel{\int_{⑧} \vec{B}\cdot d\vec{l}}^{\,0}$$

Consider one of the radial sections (2, 4, 6, 8)

For these segments: $\vec{B} \perp d\vec{l}$

$$\therefore \vec{B} \cdot d\vec{l} = 0 \text{ in this case.}$$

Next, consider circular arc paths (1, 3, 5, 7)

In these cases $\vec{B} \parallel d\vec{l}$

$$\vec{B} \cdot d\vec{l} = B\,dl$$

& $\vec{B}$ is const. in magnitude along these sections

$$\int_{1} \vec{B} \cdot d\vec{l} = \int_{1} B\,dl = B \underbrace{\int_{1} dl}_{\text{length of arc 1}}$$

$$r_2 \theta_1$$

$$\int_{1} \vec{B} \cdot d\vec{l} = B\,(r_2 \theta_1)$$

But for path ① we know $B = \dfrac{\mu_0 I}{2\pi r_2}$

$$\therefore \int_{①} \vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{2\pi \cancel{r_2}} (\cancel{r_2}\theta_1) = \frac{\mu_0 I}{2\pi}\theta_1$$

Likewise $\displaystyle\int_{③} \vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{2\pi}\theta_3$ same for ⑤ & ⑦

$$\therefore \oint \vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{2\pi}\theta_1 + 0 + \frac{\mu_0 I}{2\pi}\theta_3 + 0$$

$$+ \frac{\mu_0 I}{2\pi}\theta_5 + 0 + \frac{\mu_0 I}{2\pi}\theta_7 + 0$$

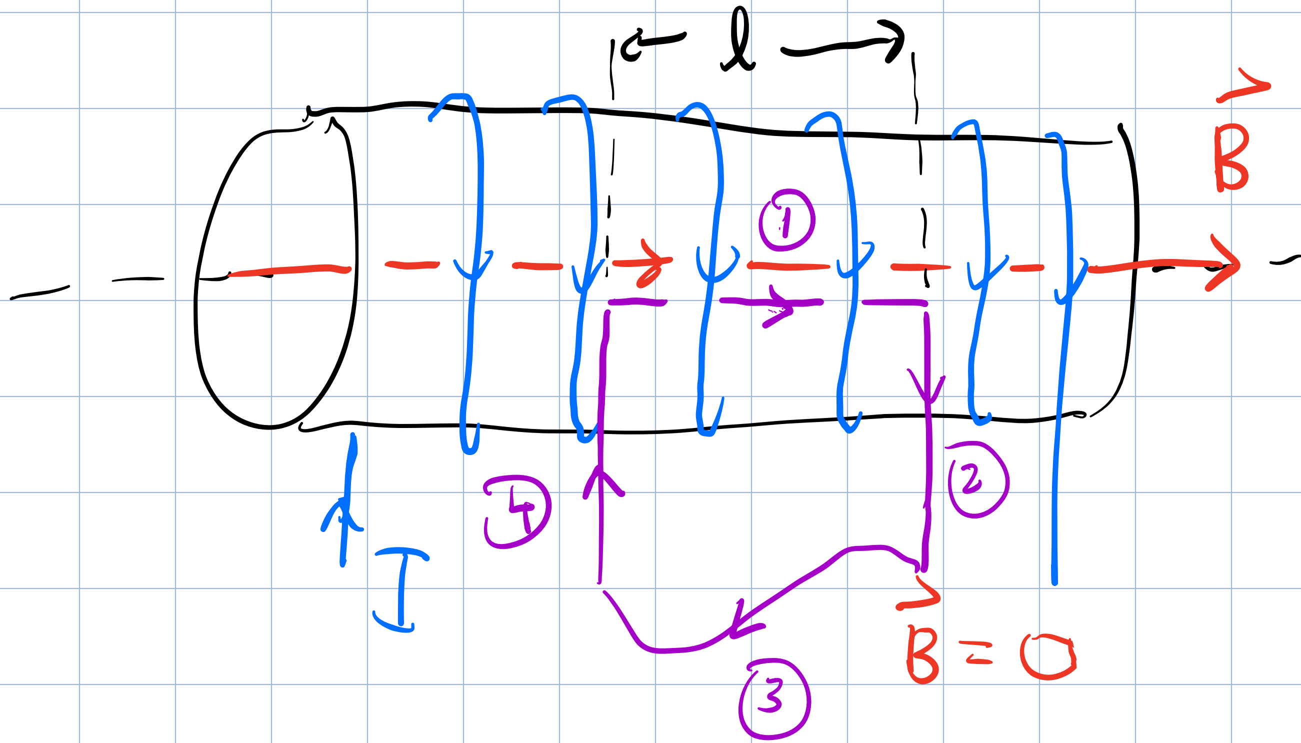
I_{encl}

$$= \frac{\mu_0 I}{\cancel{2\pi}} \Big(\theta_1 + \underbrace{\cancel{\theta_3 + \theta_5 + \theta_7}}_{2\pi}\Big)$$

$$\boxed{\therefore \oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}}$$

Ampere's Law for arbitrary path.

Apply Ampere's Law to a Solenoid



From Lab#7, know $\vec{B} \neq 0$ inside solenoid & it is parallel to solenoid axis

$\vec{B} = 0$ outside the solenoid.

To apply ampere's law, select an integration path. Want $\vec{B}$ to be $\perp$ or $\parallel$ to every segement of integration path. (purple path)

Segment 1 parallel to $\vec{B}$

② $\perp$ to $\vec{B}$ (inside) or $\vec{B} = 0$ (outside)

④ " " " " " " "

③ $\vec{B} = 0$ everywhere.

$$\oint \vec{B}\cdot d\vec{l} = \int_{①} \vec{B}\cdot d\vec{l} + \cancelto{0}{\int_{②} \vec{B}\cdot d\vec{l}} + \cancelto{0}{\int_{③} \vec{B}\cdot d\vec{l}} + \cancelto{0}{\int_{④} \vec{B}\cdot d\vec{l}}$$

$\vec{B} \perp d\vec{l}$ or $\vec{B} = 0$

since $B = 0$

$\vec{B} \perp d\vec{l}$ $\vec{B} = 0$

$$= \int_{①} B\,dl = B \underbrace{\int_{①} dl}_{l}$$

$$\therefore \oint \vec{B}\cdot d\vec{l} = Bl = \mu_0 \underbrace{I_{encl}}_{NI}$$

In general, the blue wire/current passes through purple amperian loop N times.

$\therefore$ $I_{encl} = NI$

$\therefore \quad B\ell = \mu_0 N I$

$B = \mu_0 \left(\frac{N}{\ell}\right) I$ magnetic field due to a solenoid (Lab #7).