PHYS 121 April 10, 2024

✓ The last PrairieLearn HW is due tomorrow @ 23:59

✓ Complete end-of-term survey by 23:59 today to receive 0.5 towards final grade.
Link to Survey is on PHYS 121 Canvas home-page.

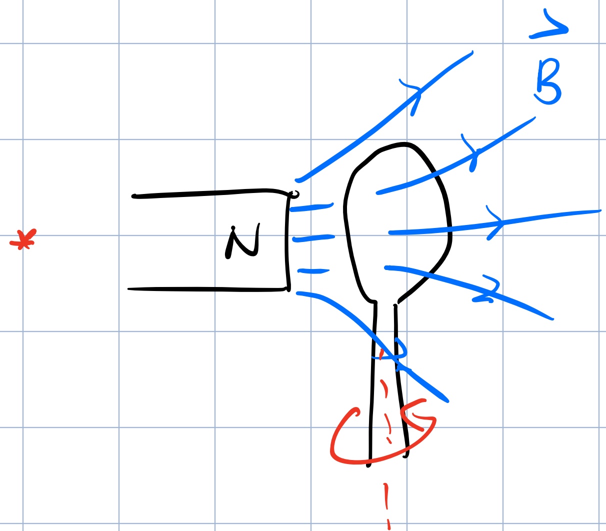
Last Time:

Faraday's Law:

$$\mathcal{E} = \left| \frac{d\Phi_B}{dt} \right| \quad *$$

$$\Phi_B = \int \vec{B} \cdot d\vec{a}$$

$= BA\cos\theta$ for flat surfaces & uniform $\vec{B}$.



Can get induce emf in three different ways.

- ✓ change the strength of $\vec{B}$
- ✓ change the loop area A (eg. motional emf)
- ✓ change θ (angle between area & $\vec{B}$)

$\Rightarrow$ electric generators.

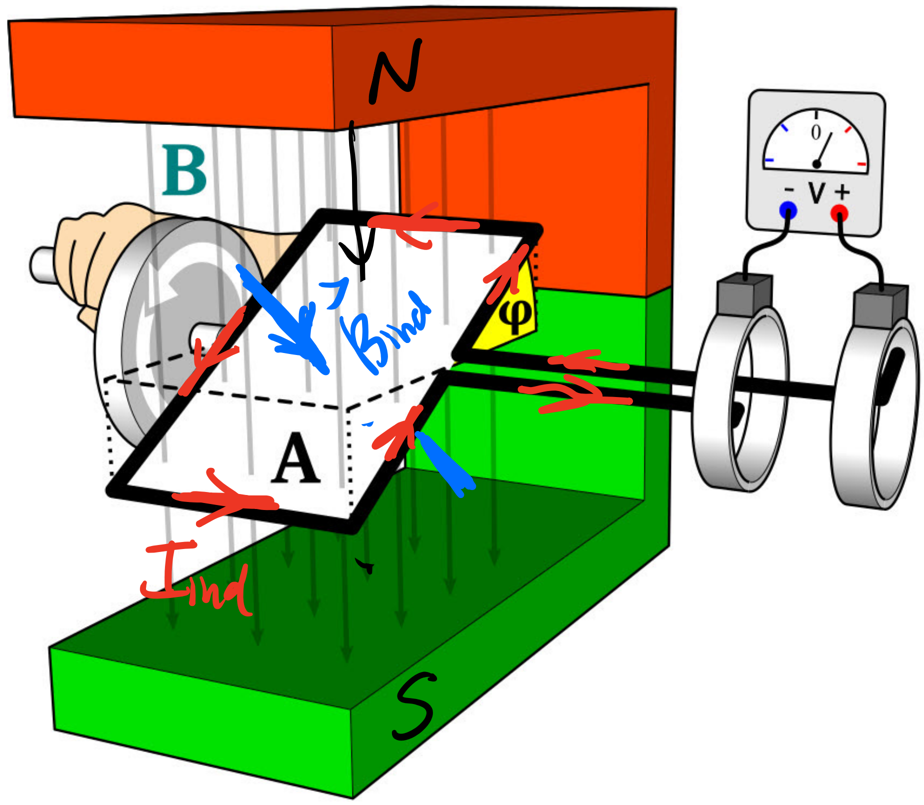
Lenze's Law:

The induced magnetic field $\vec{B}_{ind}$ has a dir'n that tends to maintain the original magnetic flux.

- If Φ_B is increasing,
  $\vec{B}_{ind}$ is in opp. dir'n of $\vec{B}$.

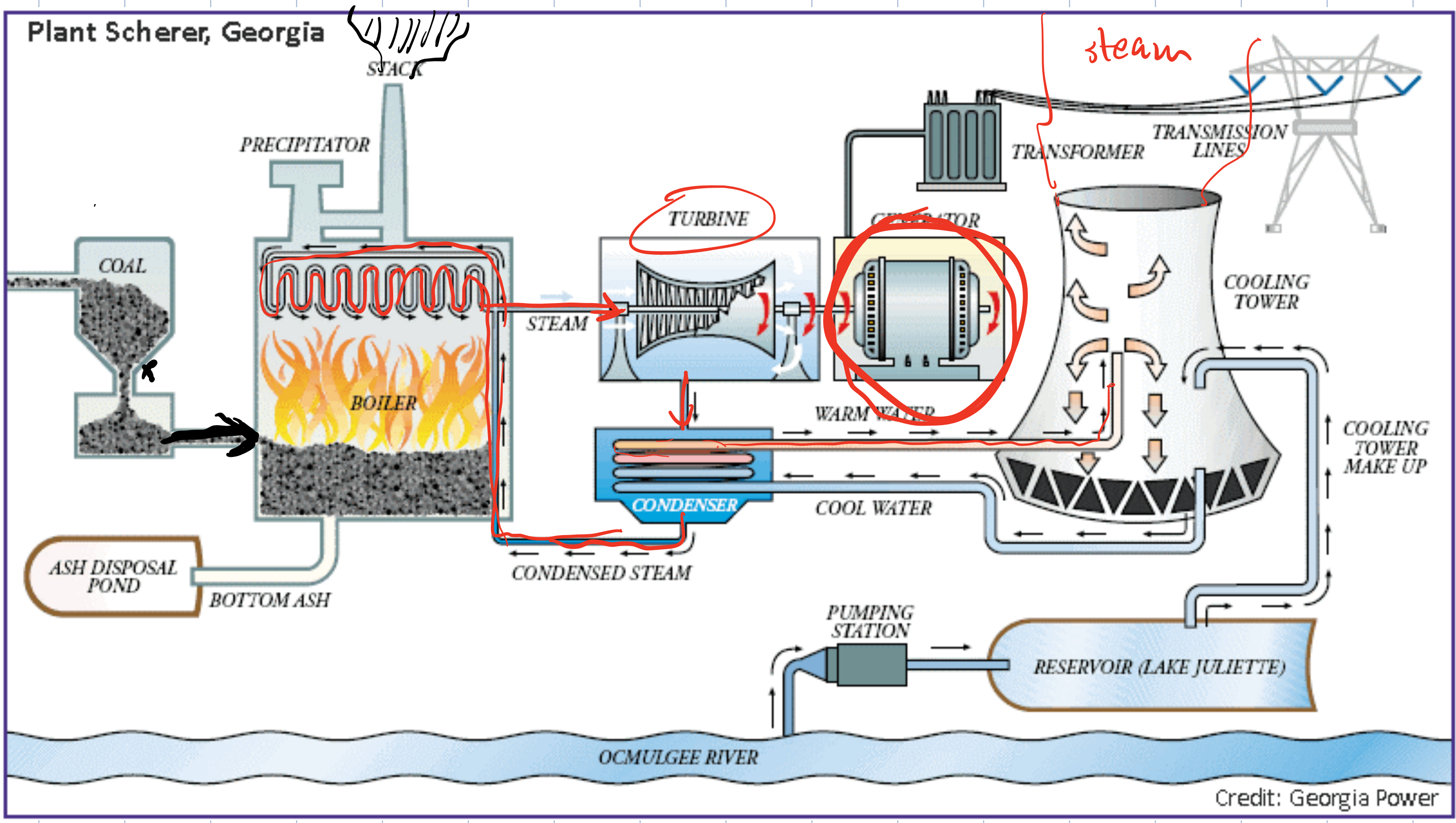
- If Φ_B is decreasing,
  $\vec{B}_{ind}$ is in same dir'n as $\vec{B}$.

Basic Electric generator - rotate a coil through a magnetic field.

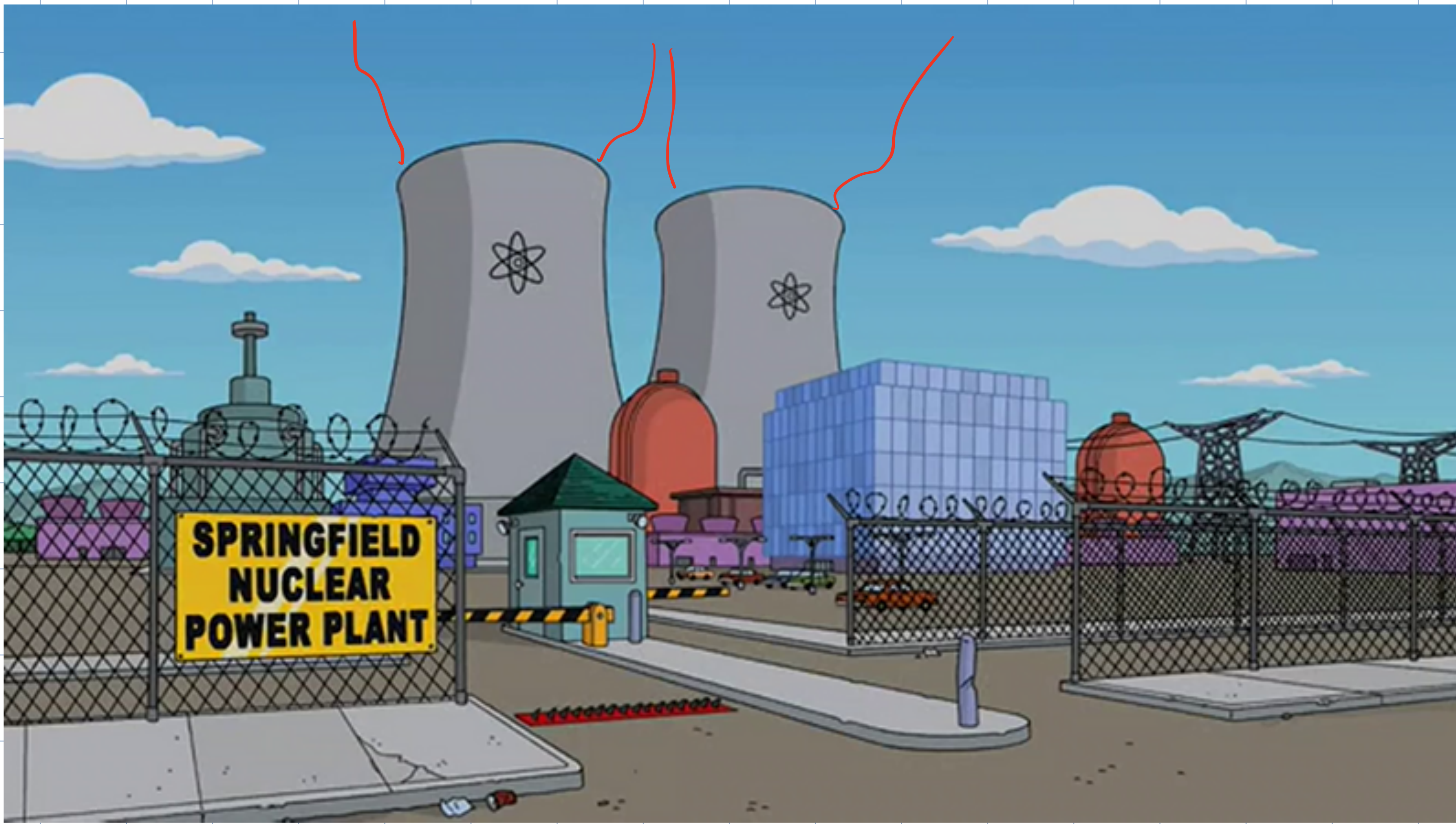


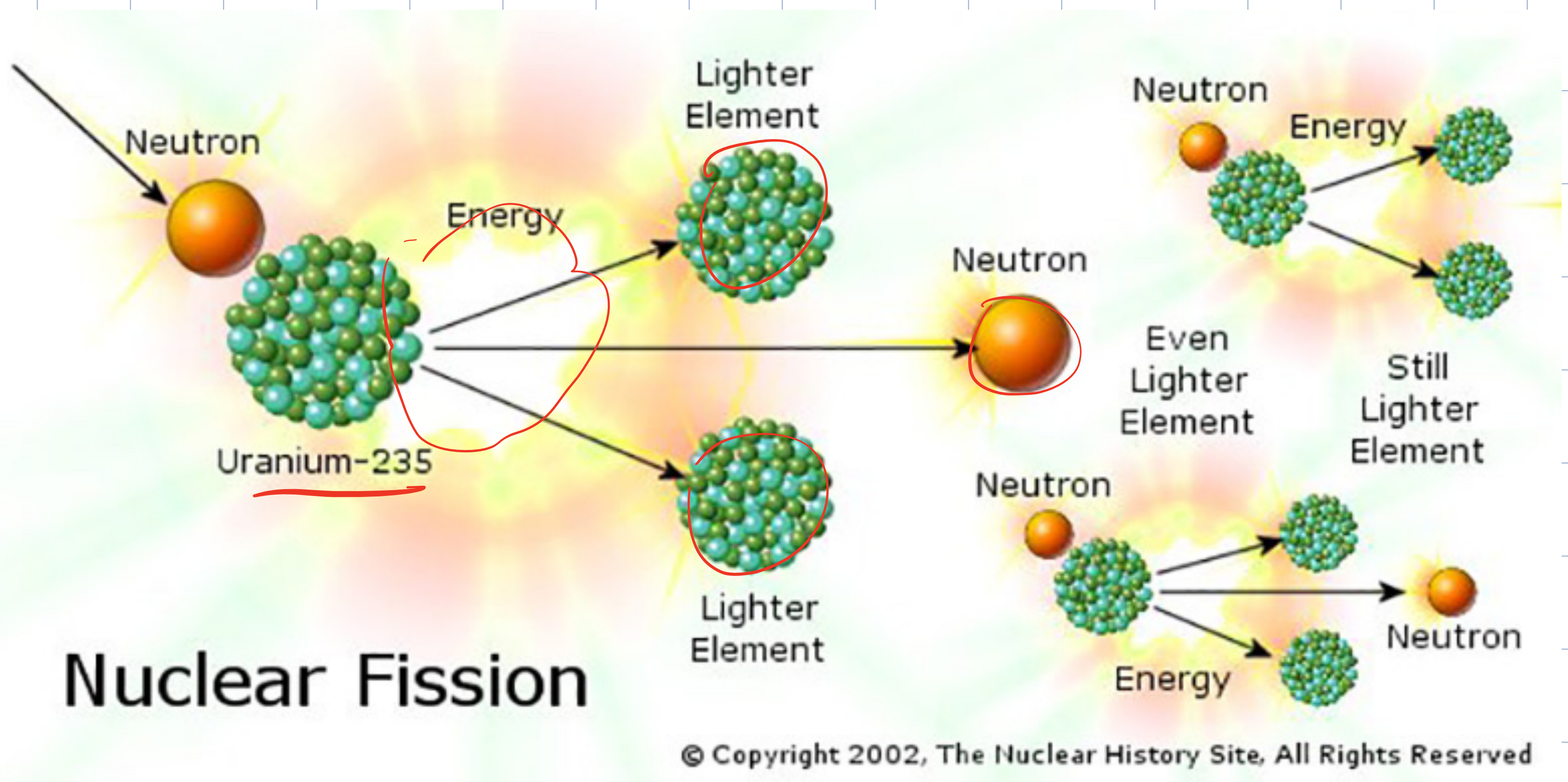
Coal Plant

Coal Plant Schematic

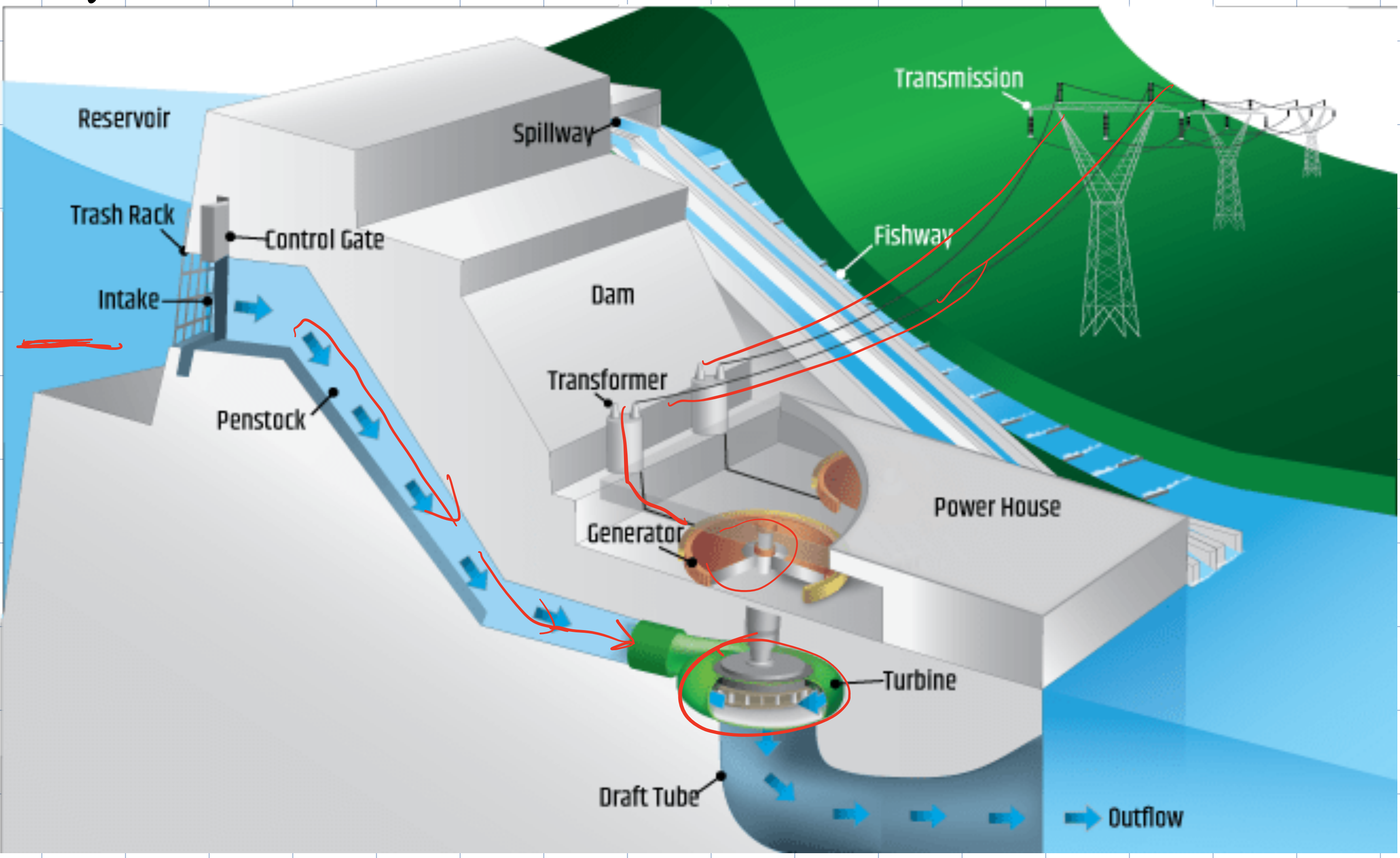


Nuclear Power Plant (Simpsons)

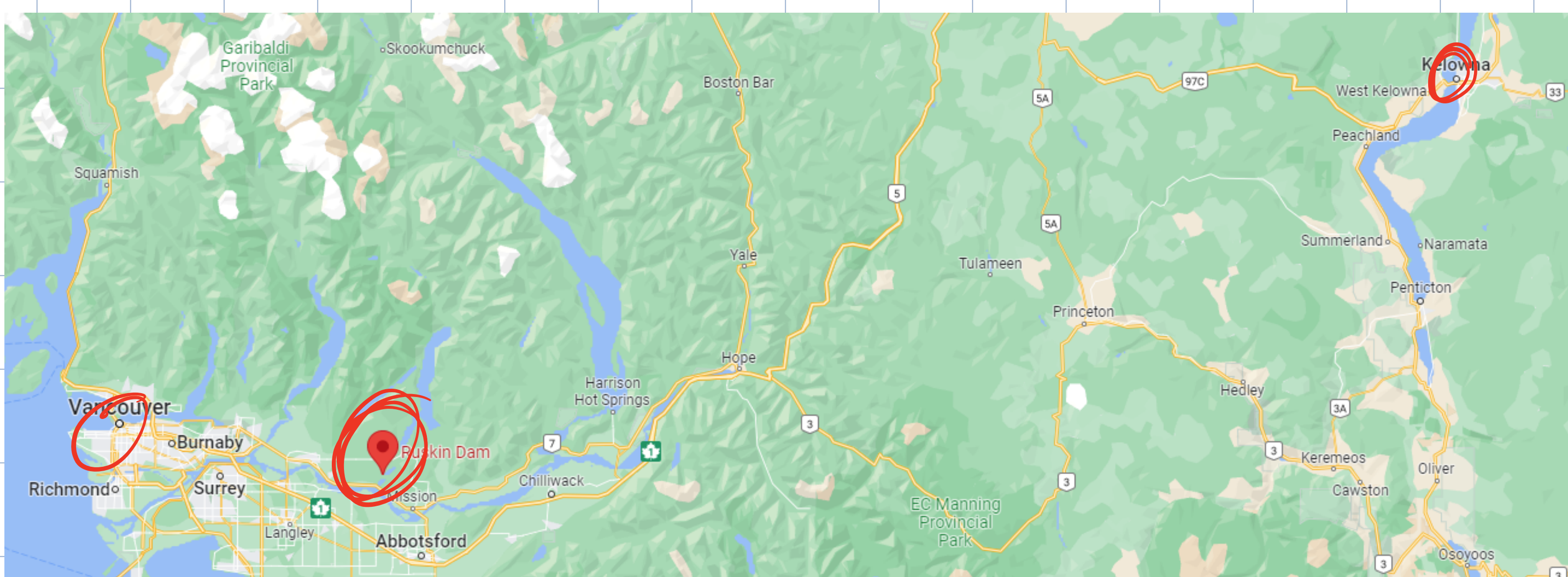




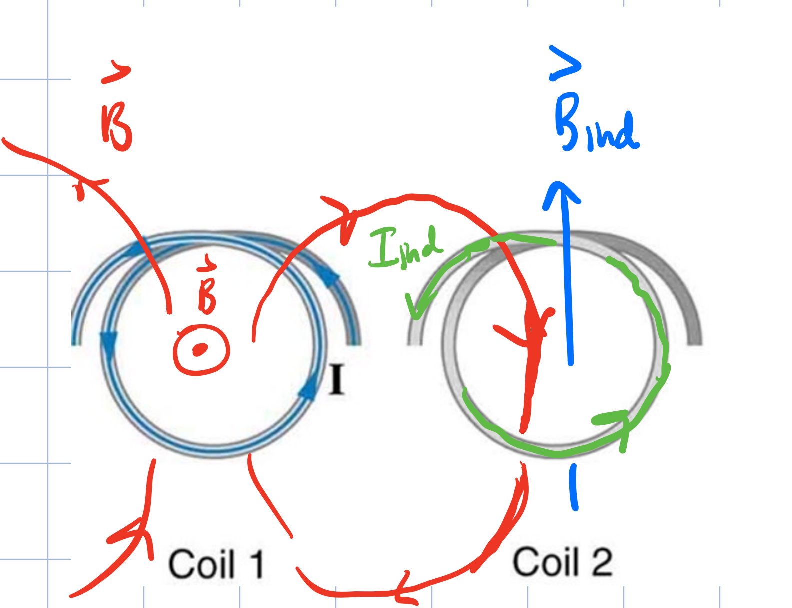
Hydro damn Schematic



Ruskin Dam. Between Maple Ridge & Mission



1: Referring to the figure, what is the direction of the current induced in coil 2: (a) If the current in coil 1 increases? (b) If the current in coil 1 decreases? (c) If the current in coil 1 is constant?



(a) For a loop of current, the dir'n of $\vec{B}$ is given by RHR.

$\vec{B}$ due to coil 1 is downwards through coil 2.

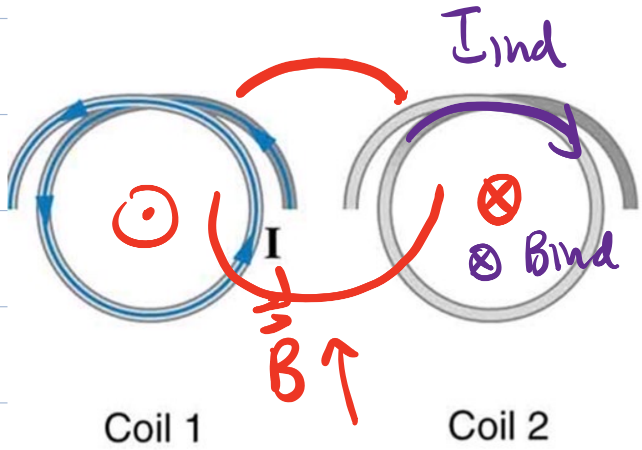
If I is increasing, then Φ through 2 is increasing in downwards dir'n. By Lenzes law $\vec{B}_{ind}$ in 2 is upwards to oppose the increase in Φ_B.

(c) If I is constant, then $\vec{B}$ is const & Φ_B through 2 is constant.

Since

$$\varepsilon = \left| \frac{d\Phi_B}{dt} \right| = 0$$

In this $\varepsilon = 0$, $I_{ind} = 0$, $\vec{B}_{ind} = 0$.

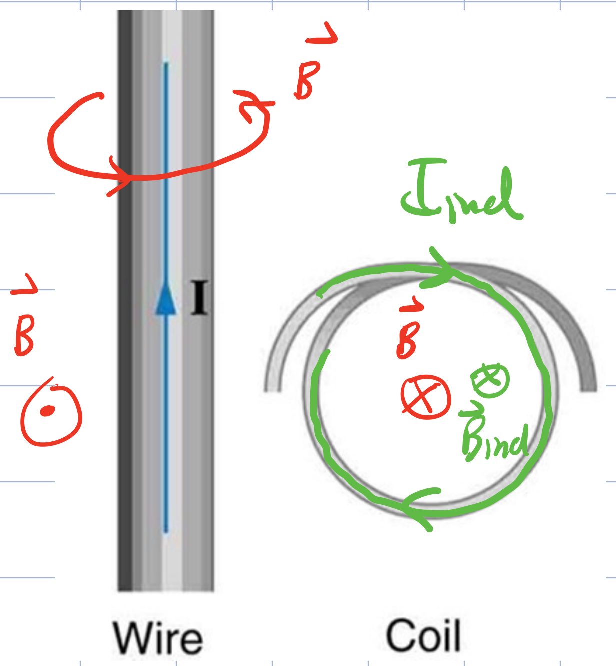


(b) If I is decreasing, then $\vec{B}$ & Φ_B through coil 2 are decreasing.

$\therefore$ $\vec{B}_{ind}$ is in same dir'n as $\vec{B}$ (into page).

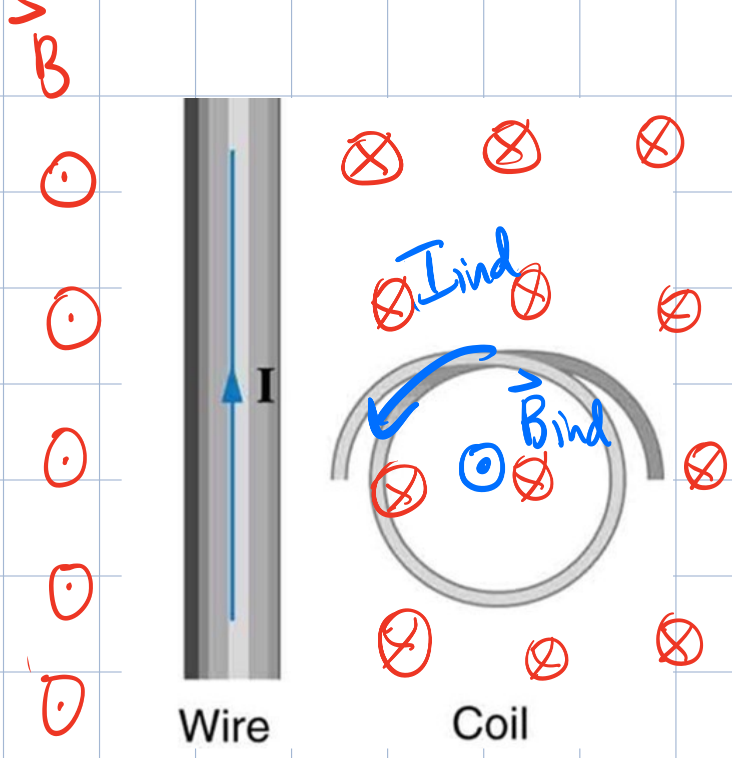
By RHR I_{ind} is CW

2: Referring to the figure, what is the direction of the current induced in the coil: (a) If the current in the wire increases? (b) If the current in the wire decreases? (c) If the current in the wire suddenly changes direction?



(b) If I is decreasing, then Φ_B through the coil is decreasing. $\vec{B}_{ind}$ is in same dir'n as $\vec{B}$ in order to try to maintain the original flux.

$\vec{B}_{ind}$ into the screen requires a CW I_{ind}.

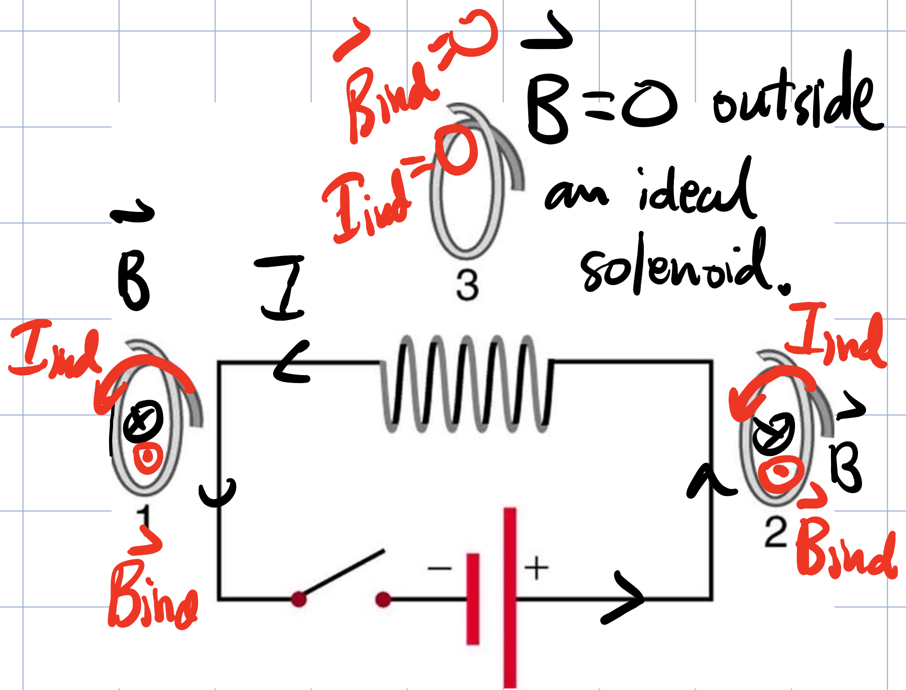


(a) If I is increasing, then $\vec{B}$ & Φ_B through coil are also increasing.

$\therefore$ $\vec{B}_{ind}$ is in opp dir'n of $\vec{B}$ (i.e. $\vec{B}_{ind}$ is out of page). By RHR I_{ind} is CCW.

(c) If current reverses dir'n, then $\vec{B}_{ind}$ will oppose the change in Φ_B. Initially, flux was directed into the page. To try to preserve that flux $\vec{B}_{ind}$ is into page & I_{ind} in CW. (Same as in part (b)).

3: Referring to the figure, what are the directions of the currents in coils 1, 2, and 3 (assume that the coils are lying in the plane of the circuit): (a) When the switch is first closed? (b) When the switch has been closed for a long time? (c) Just after the switch is opened?



(a) just after switch is closed, est. a current that is CCW in circuit.

In coils 1 & 2, $\vec{B}$ is into page & increasing (since I was initially zero. $\vec{B}_{ind}$ is in opp dir'n of $\vec{B}$ & I_{ind} is CCW.

For an ideal solenoid, $\vec{B} = 0$ outside the bore. ∴ flux through coil 3 is zero in all cases $\Rightarrow$ $\vec{B}_{ind} = 0$ & $I_{ind} = 0$

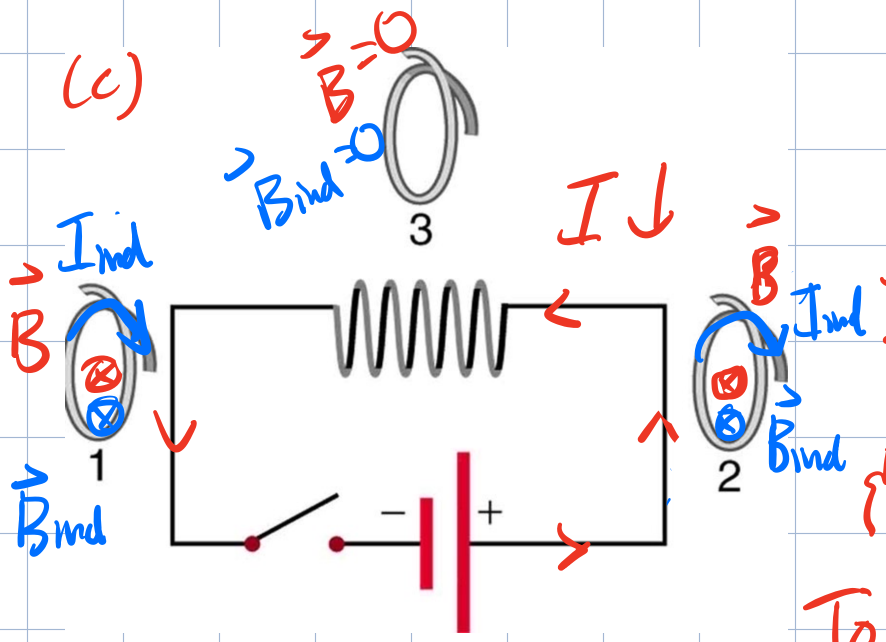
(b) If switch has been closed for a long time, then I is const & $\vec{B}$ through coils 1 & 2 is const.

$\therefore$ Φ_{B1} & Φ_{B2} are const & $\mathcal{E}_1 = \left|\frac{d\Phi_{B1}}{dt}\right| = 0$

$$\mathcal{E}_2 = \left|\frac{d\Phi_{B2}}{dt}\right| = 0$$

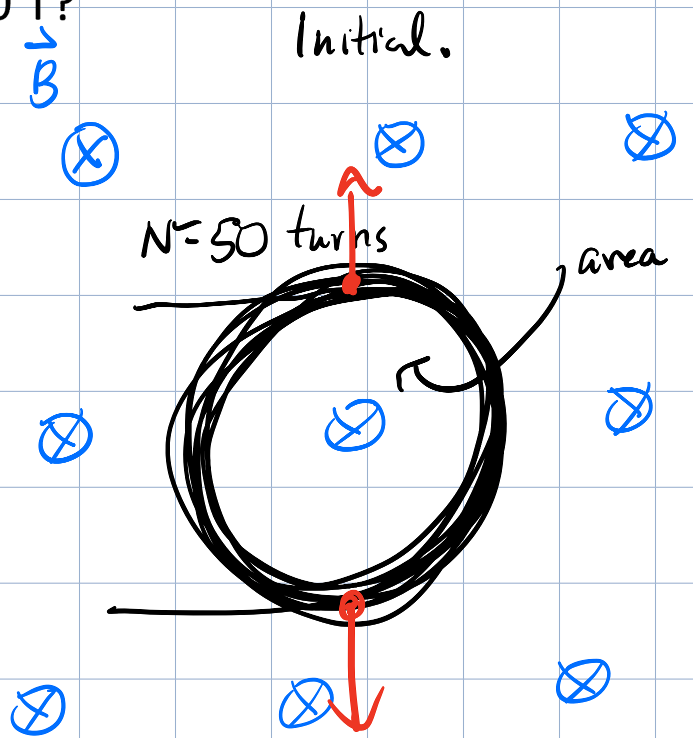
No induced emf, no induced current.

(c)



After switch is opened, I decreases such that $\vec{B}$ in coils 1 & 2 decrease & Φ_{B1} & Φ_{B2} decrease. To oppose this change in flux, $\vec{B}_{ind}$ is in same dir'n of $\vec{B}$. By RHR, I_{ind} is in CW dir'n.

4: Suppose a 50-turn coil lies in the plane of the page in a uniform magnetic field that is directed into the page. The coil originally has an area of 0.250 m². It is stretched to have no area in 0.100 s. What is the direction and magnitude of the induced emf if the uniform magnetic field has a strength of 1.50 T?

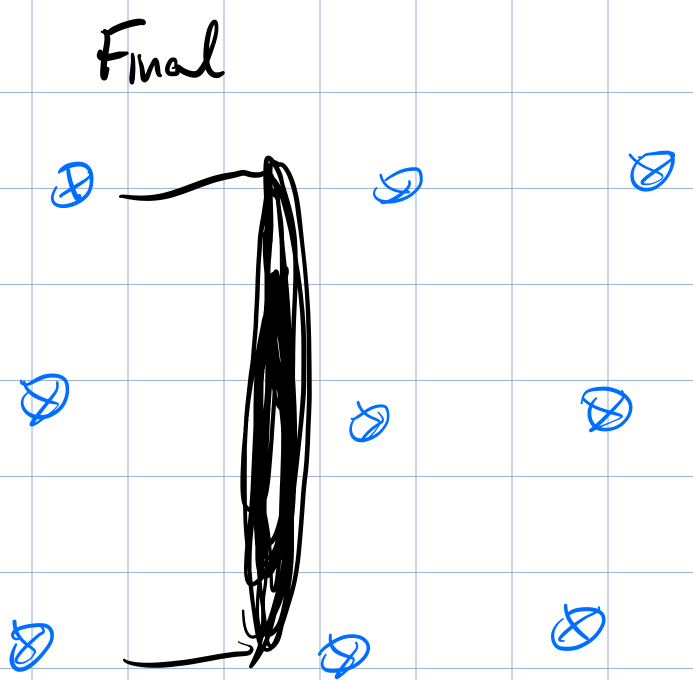


Flux through a single turn of coil

$$\Phi_1 = BA$$

Since we have N=50 turns, the total initial flux is

$$\Phi_{B_i} = N\Phi_1 = NBA$$



Now the area of the coil is zero.

$$\Phi_{B_f} = NB\cancel{A}^{\,0} = 0$$

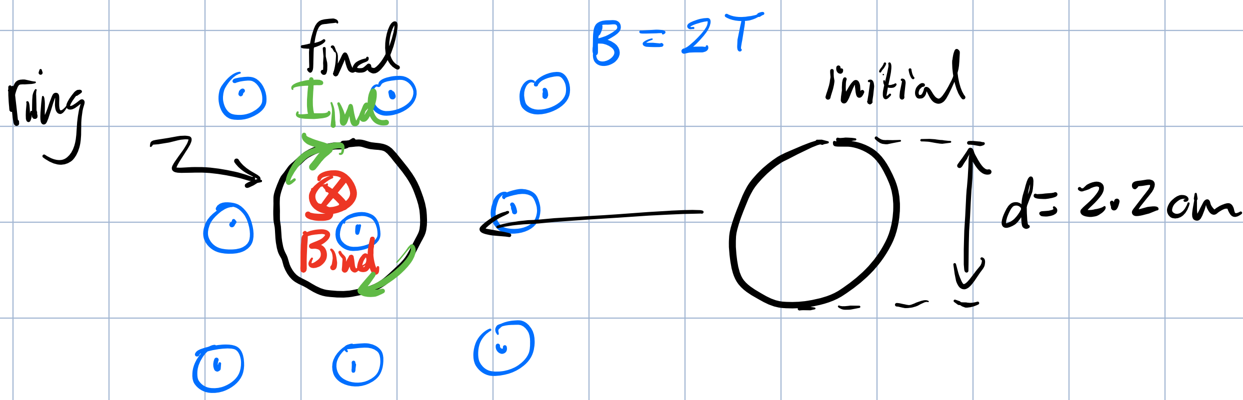
$$\Delta\Phi_B = \Phi_{B,f} - \Phi_{B,i}$$
$$= 0 - NBA = -NBA$$

$$\mathcal{E} = \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{NBA}{\Delta t}$$

$N = 50$, $B = 1.5\,T$, $A = 0.25\,m^2$, $\Delta t = 0.1\,s$

$$= \frac{750}{4}\,V = \underline{\underline{187.5\,V}}$$

5: (a) An MRI technician moves his hand from a region of very low magnetic field strength into an MRI scanner's 2.00 T field with his fingers pointing in the direction of the field. Find the average emf induced in his wedding ring, given its diameter is 2.20 cm and assuming it takes 0.250 s to move it into the field. (b) Discuss whether this current would significantly change the temperature of the ring.



Δt
$= 0.25$ s

$$(a) \quad \mathcal{E} = \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \left|\frac{\overbrace{B\pi\left(\frac{d}{2}\right)^2}^{\Phi_{B,f}} - \overbrace{0}^{\Phi_{B,i}}}{\Delta t}\right|$$

$$\therefore \quad \mathcal{E} = \frac{\pi B d^2}{4\Delta t} = 3.04 \text{ mV (small)}$$

6: Referring to the situation in the previous problem: (a) What current is induced in the ring if its resistance is 0.0100 Ω? (b) What average power is dissipated? (c) What magnetic field is induced at the center of the ring? (d) What is the direction of the induced magnetic field relative to the MRI's field?

(a) Find I. $I = \frac{\mathcal{E}}{R} = \frac{B\pi d^2}{4\Delta t R}$

$B = 2\,T$

$d = 2.2\,cm = 2.2\times10^{-2}\,m$ $\quad I = 0.3\,A$

$\Delta t = 0.25\,s$

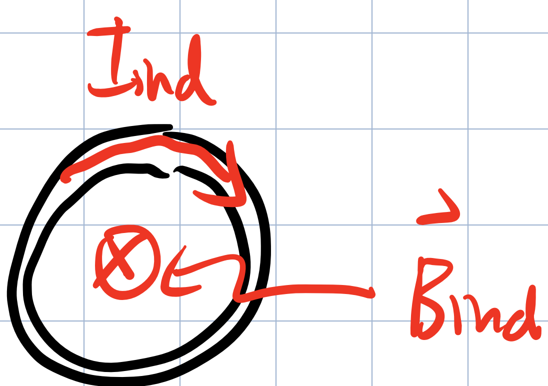
$R = 0.01\,\Omega$

(b) $P = I^2 R = (0.09)(0.010)$

$= 9\times10^{-4}\,W$

$= 0.9\,mW$ (small)

(c)



$B_{ring} = \frac{\mu_0 I}{2r}$

$I = 0.3A$

$B_{ring} = \dfrac{\mu_0 I}{d}$

$\mu_0 = 4\pi \times 10^{-7}$

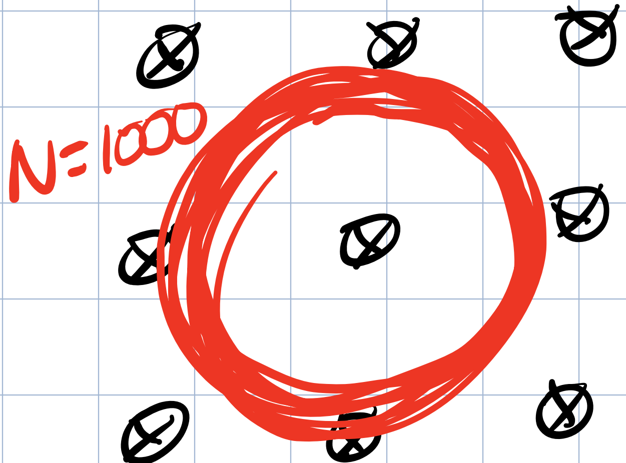
$d = 2.2 \times 10^{-2} m$

(d) $\vec{B}_{ind}$ is in <u>opp.</u> dir'n as $\vec{B}$ from MRI scanner by Lenz's law.

7: An emf is induced by rotating a 1000-turn, 20.0 cm diameter coil in the Earth's 5.00 × 10^{-5} T magnetic field. What average emf is induced, given the plane of the coil is originally perpendicular to the Earth's field and is rotated to be parallel to the field in 10.0 ms?

initially

$\vec{B}_{Earth} = 50\mu T$

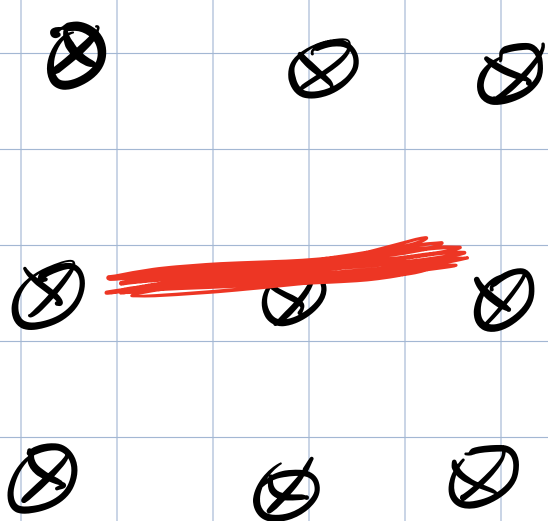


$\Phi_{B,i} = NB\pi\left(\frac{d}{2}\right)^2$

each turn of the coil has flux of $B\pi\left(\frac{d}{2}\right)^2$

$\Delta t = 10ms$

final



$\Phi_{B,f} = 0$

$$\Delta\Phi_B = \cancelto{0}{\Phi_{B,f}} - \Phi_{B,i} = -NB\pi\left(\frac{d}{2}\right)^2$$

$$\mathcal{E} = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{NB\pi\left(\frac{d}{2}\right)^2}{\Delta t}$$

using $N = 1000$

$B = 50\,\mu T$

$d = 0.20\,m$

$\Delta t = 0.010\,s$

find $\underline{\mathcal{E} = 0.157\,V}$

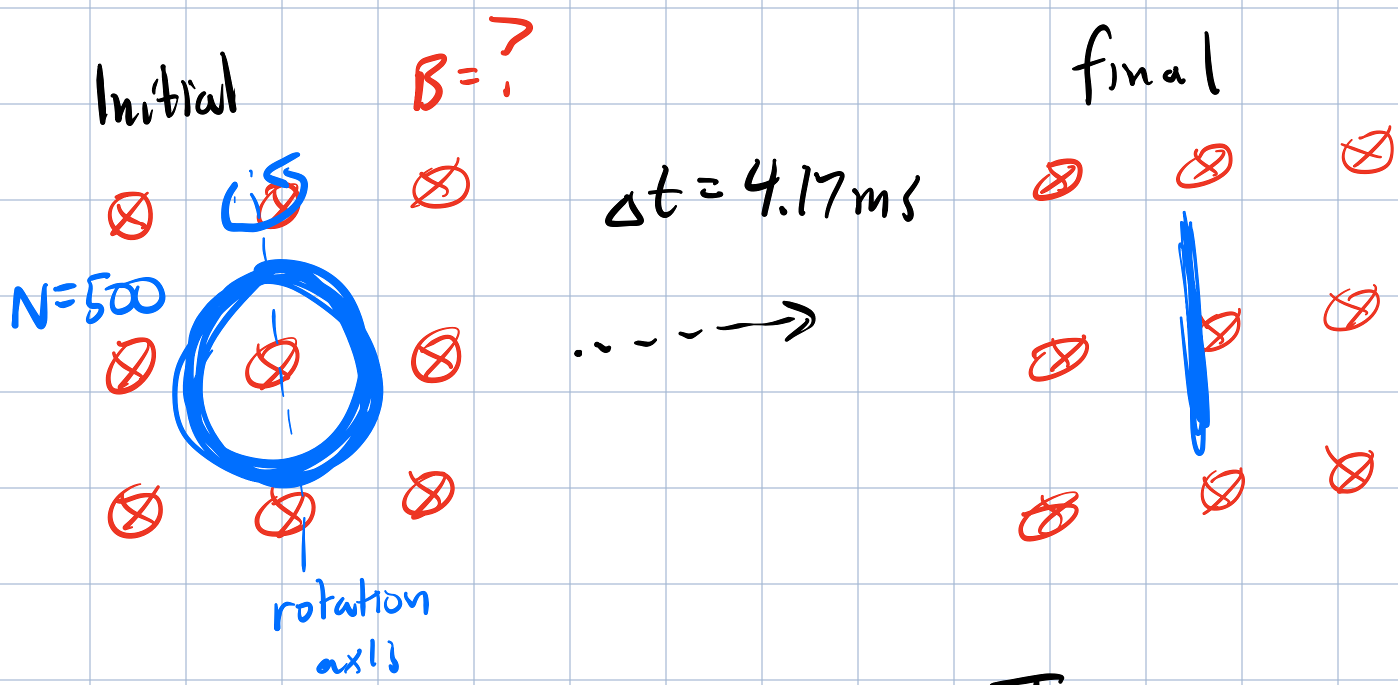
8: A 0.250 m radius, 500-turn coil is rotated one-fourth of a revolution in 4.17 ms, originally having its plane perpendicular to a uniform magnetic field. (This is 60 rev/s.) Find the magnetic field strength needed to induce an average emf of 10,000 V.

$$\frac{1}{4} \text{ rev} = 90^\circ \text{ or } \frac{\pi}{2} \text{ rad.}$$

$r = 0.250 \text{ m}$

$N = 500$

$\Delta t = 4.17 \times 10^{-3} \text{ s}$



$\Phi_{B,f} = 0$

$$\Phi_{B,i} = N\underbrace{B\pi r^2}_{\text{flux in each loop of the coil}}$$

$$\Delta\Phi_B = \Phi_{B,f} - \Phi_{B,i} = -NB\pi r^2$$

$$\varepsilon = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{NB\pi r^2}{\Delta t}$$

want $\varepsilon = 10\times10^3 V$
$= 10 kV$

solve for B

$$B = \frac{\varepsilon\,\Delta t}{N\pi r^2} = \boxed{0.425\,T}$$