

Irving K. Barber School
of Arts and Sciences
UBC Okanagan

Date: Feb 1st, 2015 Time: 11:30am Duration 35 minutes.
This exam has 6 questions for a total of 28 points.

## SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

| Problem | Points <br> Earned | Points <br> Out Of |
| :---: | :---: | :---: |
| 1 |  | 7 |
| 2 |  | 7 |
| 3 |  | 5 |
| 4 |  | 5 |
| 5 |  | 4 |
| TOTAL: |  | $\mathbf{2 8}$ |

CANDIDATE NAME (print): $\qquad$

STUDENT NUMBER: $\qquad$

Signature: $\qquad$

7 1. An auditorium is 100 m in length, 50 m in width, and 30 m in height. It is ventilated by a system that feeds in fresh air and draws out air at the same rate. If the auditorium air is well-mixed, what inflow (and outflow) rate is required to reduce any air pollutants present by a factor of 100 in 30 minutes? Include a diagram in your solution.
2. Use the given direction field to answer the questions below.


2 (a) Draw several solution curves, some starting at $y=3$, some starting at $y$ just above zero, and some starting at $y$ just below zero.
3 (b) What type of ODE produced this direction field? Write a plausible guess for what this ODE is;
(c) What can you say about the solution trajectories as $t \rightarrow-\infty$ ?

5 3. Find the solution to the initial value problem

$$
y^{\prime}+2 t y=2 t e^{-t^{2}}, \quad y(1)=2
$$

5 4. Find an integrating factor for the equation

$$
\left(3 x y+y^{2}\right) d x+\left(x^{2}+x y\right) d y=0 .
$$

Make sure you verify that your new equation is indeed exact! (Hint: The integrating factor is a function of $x$.)

4 5. Use the Forward Euler method to approximate the solution to the IVP below using steps of size $h=0.1$. Enter your results in the table provided. Show your calculations below.

$$
\frac{d v}{d t}=\frac{t}{v}, \quad v(0)=-1
$$

| $n$ | $t_{n}$ | $v_{n}$ | $v_{n+1}$ |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| 0 | 0 |  |  |
| 1 |  |  |  |
|  |  |  |  |
|  |  |  |  |

6. BONUS PROBLEM for the Group Test (you can start working on this problem while waiting for the group test to start): Solve (implicitly) the exact differential equation

$$
\left(\sin (x)+x^{2} e^{y}-1\right) d y+\left(y \cos (x)+2 x e^{y}\right) d x=0 .
$$

