

Irving K. Barber School
of Arts and Sciences
UBC Okanagan

Date: Mar 16th, 2016 Time: 12:05pm Duration 20 minutes.
This exam has 5 questions for a total of 22 points.

## SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

| Problem | Points <br> Earned | Points <br> Out Of |
| :---: | :---: | :---: |
| 1 |  | 3 |
| 2 |  | 3 |
| 3 |  | 7 |
| 4 |  | 9 |
| BONUS |  | 2 |
| TOTAL: |  | $\mathbf{2 2}$ |

CANDIDATE NAMES (print): $\qquad$

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3 1. Determine the form of a particular solution to the ODE below. Do not solve for the coefficients!

$$
y^{\prime \prime}+4 y^{\prime}+5 y=e^{2 t}+t^{2} \sin (t)-e^{2 t} \cos (t)
$$

2. a mass weighing 8 kg is attached to a spring with stiffness constant $10 \mathrm{~N} / \mathrm{m}$. At $t=0$, the mass is at it's equilibrium position, and an external force $F(t)=2 \cos (2 t) \mathrm{N}$ is applied to the system. The damping constant for the system is $1 \mathrm{Ns} / \mathrm{m}$.
(a) Write the IVP that governs the system.

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(b) What is the resonance frequency of the system? (give the exact answer)

7 3. Consider the ODE $y^{\prime \prime}+y=\tan (t)$. Given that $y_{h}(t)=c_{1} \cos (t)+c_{2} \sin (t)$, find a particular solution.
(Hint: You may find it useful to know that $\int \sec (u) d u=\ln |\sec (u) \tan (u)|+C$.)

9 4. Consider the following forced spring-mass system:

$$
4 y^{\prime \prime}+4 y^{\prime}+5 y=17 \cos (t), \quad y(0)=y^{\prime}(0)=0
$$

Given that

$$
y_{h}(t)=e^{-\frac{1}{2} t}\left(c_{1} \cos (t)+c_{2} \sin (t)\right),
$$

find the equation of motion of the mass.

Workspace for question 4.
5. BONUS problem for the Group Test - 2 points Use the mass-spring analogy to determine the qualitative form of solutions to the IVP

$$
U^{\prime \prime}+c U^{\prime}+U(1-U)=0, \quad U(0)=0.9, \quad U^{\prime}(0)=0
$$

where $c$ is an arbitary positive constant. (Note: This equation is known as the FisherKolmogorov equation.)

