a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

Irving K. Barber School
of Arts and Sciences
UBC Okanagan

Instructor: Rebecca Tyson Course: MATH 225
Date: Feb 6th, 2017 Time: 11:30am Duration: 35 minutes.
This exam has 4 questions for a total of 23 points.

UBC ID #: _____ NAME (print): _____

Signature: _____

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

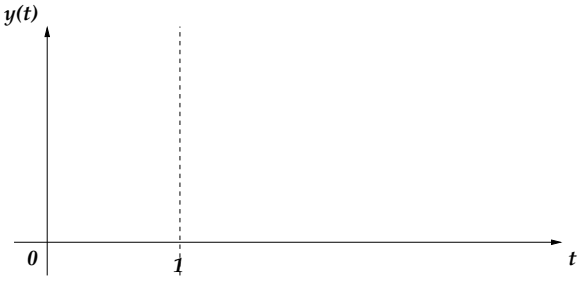
This is a two-stage exam. You have 45 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 35 minutes.

Question:	1	2	3	4	Total
Points:	7	3	6	7	23
Score:					

1. Consider the initial value problem

$$\frac{dy}{dt} = \frac{2}{y}(1-t), \qquad y(0) = y_0 > 0.$$

2 (a) On the axes below, sketch a few arrows (about half a dozen) to show the general shape of the direction field.



5 (b) Solve the initial value problem, and sketch the solution on the direction field above.

3 2. Find the most general function $M(x, y)$ so that the equation below is exact:

$$M(x, y)dx + \left(\sec^2(y) - \frac{x}{y}\right) dy = 0.$$

6 3. Find the general solution to the ODE

$$x\frac{dy}{dx} + 3(y + x^2) = \frac{\sin(x)}{x}.$$

4. Consider the ODE

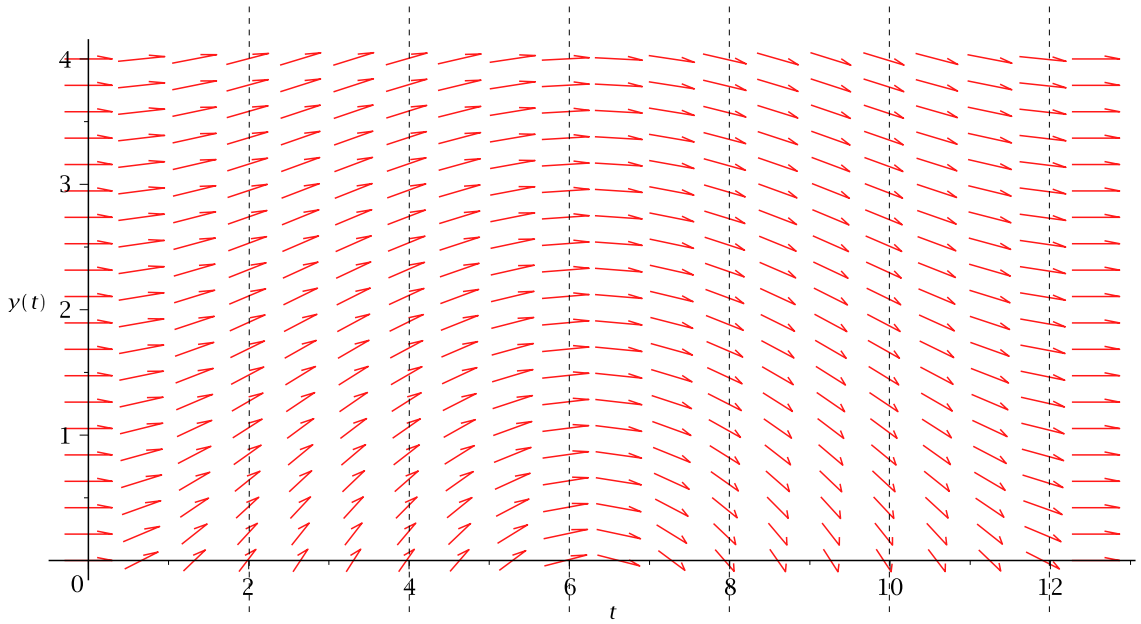
$$\frac{dy}{dt} = \frac{1}{y-1}\sin\left(\frac{t}{2}\right). \tag{1}$$

[2] (a) Write the Forward Euler formula for (1) using timesteps of size h.

(b) The direction field corresponding to (1) is given below. On that direction field, draw two curves starting at $(t, y) = (0, 1)$:

[1] i. Sketch the true solution, following the direction field "by eye."

[2] ii. Using a ruler, carefully draw the Forward Euler solution, using a stepsize of $h = 2$.



[2] (c) Does the Forward Euler solution underestimate or overestimate the true solution?