UBC ID #: ________________ NAME (print): ______________________________

Signature: Solutions

a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

IRVING K. BARBER SCHOOL
OF ARTS AND SCIENCES
UBC OKANAGAN

Instructor: Rebecca Tyson Course: MATH 225
Date: Mar 22nd, 2017 Time: 11:30am Duration: 35 minutes.
This exam has 4 questions for a total of 501 points.

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is a two-stage exam. You have 35 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 35 minutes.

Question:	1	2	3	4	5	Total
Points:	3	2	6	6	3	20
Score:						

$\boxed{3}$ 1. Find the general solution of $t^2z'' + tz' + 9z = 0$.

$z = t^r$, characteristic eqn

$r(r-1) + r + 9 = 0 \Leftrightarrow r^2 - r + r + 9 = 0$

$\Leftrightarrow r^2 + 9 = 0$

$\Leftrightarrow r = \pm 3i$

$\therefore z(t) = c_1 \cos(3\ln(t)) + c_2 \sin(3\ln(t))$

$\boxed{2}$ 2. For which of the ODEs below could you use the method of undetermined coefficients (MoUC) to find a particular solution? In cases where MoUC applies, give the form of the particular solution.

(a) $4y'' + ty = 2cos(t)$,

This is the only case where MoUC applies. → (b) $y'' + 3y' - y = t\cos(2t)$

(c) $y'' - 2y' + y = \dfrac{e^t}{1+t^2}$

$y_p = (At+B)\cos(2t) + (Ct+D)\sin(2t)$

16

~~100~~ 2. Consider the ODE $y'' - 2y' + y = e^t/t$. Given that two linearly independent solutions of the associated homogeneous ODE are $y_1(t) = e^t$ and $y_2(t) = te^t$, find a general solution of the ODE. $\boxed{t>0}$

$y_p(t) = v_1 e^t + v_2 te^t$

$v_1(t)$ & $v_2(t)$ must satisfy

$$\begin{cases} v_1' e^t + v_2' te^t = 0 \\ v_1' e^t + v_2'(e^t + te^t) = \dfrac{e^t}{t} \end{cases} \iff$$

$$\iff \begin{cases} v_1' + v_2' t = 0 \\ v_1' + v_2'(1+t) = \dfrac{1}{t} \end{cases} \iff \begin{cases} v_2' = \dfrac{1}{t} \\ v_1' = -tv_2' = -1 \end{cases}$$

$$\begin{cases} v_2 = \ln(t) + \cancel{c}^{\to 0} \\ v_1 = -t + \cancel{c}^{\to 0} \end{cases}$$

not linearly independent of $y_2(t)$

$\therefore y_p(t) = -te^t + \ln(t) te^t$

$\therefore y(t) = c_1 e^t + c_2 te^t + \ln(t) te^t$

16 $\boxed{\cancel{100}}$ 3. Consider the ODE $y'' - 4y' + 4y = 0$. The characteristic equation has a double root, $r = 2$, and so one solution of the ODE is $y_1(t) = e^{2t}$. Use reduction of order to derive a second linearly independent solution. Write the general solution.

Let $y_2(t) = v(t)\,y_1(t) = v(t)e^{2t}$

Then $y_2' = v'e^{2t} + 2ve^{2t}$

$y_2'' = v''e^{2t} + 4v'e^{2t} + 4ve^{2t}$

The ODE becomes

$$y_2'' - 4y_2' + 4y_2 = 0 \iff v''e^{2t} + \cancel{4v'e^{2t}} + \cancel{4ve^{2t}} - \cancel{4v'e^{2t}} - \cancel{8ve^{2t}} + \cancel{4ve^{2t}} = 0$$

$$\iff v'' = 0$$

$$\iff v = At + \overset{\nearrow 0}{B}$$

$\therefore\ y_2(t) = te^{2t}$

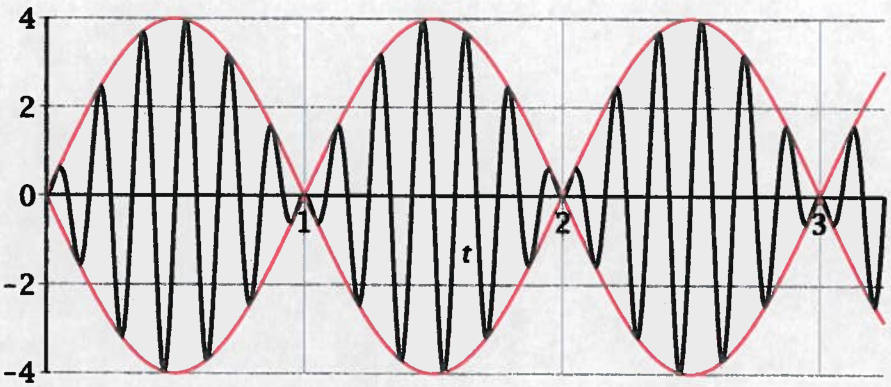
general solution:

$$y(t) = c_1e^{2t} + c_2te^{2t}$$

3 [100] 4. The solution behaviour of a particular mass-spring system is shown below. With reference to the figure, answer the following questions:

[1] (a) What is the illustrated behaviour called? (2 names)

[100] (b) What ingredients are necessary to produce this behaviour? List all of them.



a) Beats or amplitude modulation (as in AM radio). It is useful for tuning instruments, or selecting a radio station.

b) • low damping (so that the mass-spring system wants to oscillate even in the absence of forcing

• a forcing frequency of the form $F_0 \cos(\gamma t)$

• γ very close to $\omega = \sqrt{\frac{k}{m}}$ (<u>not</u> equal!)

BONUS PROBLEM, 2pts Determine the mass-spring frequency (in the absence of forcing) and the forcing frequency for the mass-spring system in question 4.

Beat period $= 2 = \frac{2\pi}{\theta} \Leftrightarrow \theta = \pi = \frac{\gamma - \omega}{2}$

"note" period $= \frac{1}{6} = \frac{2\pi}{\phi} \Leftrightarrow \phi = 12\pi = \frac{\gamma + \omega}{2}$

Solve for γ & ω:

$$\begin{cases} \gamma - \omega = 2\pi \\ \gamma + \omega = 24\pi \end{cases} \Leftrightarrow \begin{cases} 2\gamma = 26\pi \\ 2\omega = 22\pi \end{cases} \Leftrightarrow \begin{cases} \gamma = 13\pi \\ \omega = 11\pi \end{cases}$$

We don't actually know that $\gamma > \omega$. What we do know is that the forcing frequency is either 13π or 11π, & the mass-spring system wants to oscillate at the other frequency.