

UBC ID #: _____ NAME (print): _____

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a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

IRVING K. BARBER SCHOOL
OF ARTS AND SCIENCES
UBC OKANAGAN

Instructor: Rebecca Tyson Course: MATH 225

Date: Jan 31st, 2018 Time: 11:30am Duration: 35 minutes.

This exam has 5 questions for a total of 24 points.

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is a two-stage exam. You have 35 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 35 minutes.

Question:	1	2	3	4	5	Total
Points:	3	4	7	3	7	24
Score:						

- 3 1. Sketch the phase line for the ODE $y' = (y + 1)(y - 2)(y - 3)$ and state the nature of its steady states.

- 4 2. Find all solutions to the separable ODE

$$\frac{1}{\theta} \frac{dy}{d\theta} = \frac{y \sin(\theta)}{y^2 + 1}, \quad y(\pi) = 1.$$

(Possibly useful integral: $\int \theta \sin(\theta) d\theta = \sin(\theta) - \theta \cos(\theta) + C$)

3. Suppose a brine containing salt at a concentration of 0.2 kg/L runs into a tank initially filled with 500 L of water containing 5 kg of salt. The brine enters the tank at a rate of 5 L/min, and the well-stirred mixture flows out at the same rate. Let $X(t)$ be the amount of salt in the tank at time t .

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- (a) Make a sketch showing the tank, inflow, and outflow information. Write down the ODE and initial conditions for $X(t)$. Simplify the ODE.

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- (b) sketch the phase line for the ODE. What is $\lim_{t \rightarrow \infty} X(t)$?

- 3 4. Find the value of k so that the differential equation below is exact.

$$(y^3 + kxy^4 - 2x)dx + (3xy^2 + 20x^2y^3)dy = 0$$

- 7 5. Solve the initial value problem $x \frac{dy}{dx} + 3y + 2x = 3x^2$, $y(1) = 1$.