Math 225 Mock Midterm #1, Individual Test Jan ~~31st~~ 29th, 2018

UBC ID #: _____ NAME (print): _____

Signature: Solutions

a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

IRVING K. BARBER SCHOOL
OF ARTS AND SCIENCES
UBC OKANAGAN

Instructor: Rebecca Tyson Course: MATH 225
Date: Jan ~~31st~~ 29th, 2018 ~~Time: 11:30am~~ Duration: 35 minutes.
This exam has 4 questions for a total of 21 points.

**SPECIAL INSTRUCTIONS**

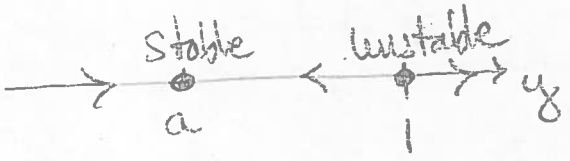
- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is a two-stage exam. You have 35 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 35 minutes.

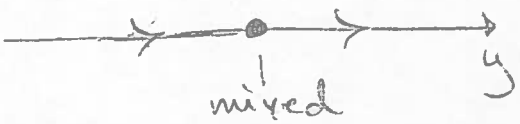
| Question: | 1 | 2 | 3 | 4 | Total |
|---|---|---|---|---|---|
| Points: | 5 | 5 | 5 | 6 | 21 |
| Score: | | | | | |

[5] 1. Consider the ODE $y' = (y-1)(y-a)$. Sketch the phase line and state the nature of its steady states. Note that your answer depends on the value of $a$. You should consider all possibilities.

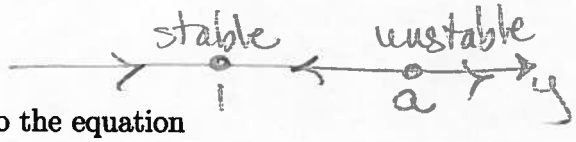
If $a < 1$:




$a = 1$:




$a > 1$:




[5] 2. Obtain the general solution to the equation

$$\frac{dy}{dx} = \frac{y}{x} + 2x + 1$$

$\frac{dy}{dx} - \frac{1}{x}y = 2x+1$ $\quad \therefore P(x) = -\frac{1}{x}$, $\quad Q(x) = 2x+1$

$\mu(x) = e^{\int -\frac{1}{x}\,dx} = e^{-\ln|x| + C \to 0} = e^{\ln \frac{1}{|x|}} = \frac{1}{|x|}$

take $\mu(x) = \frac{1}{x}$

$\therefore$ we have

$\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = 2 + \frac{1}{x} \Leftrightarrow \frac{d}{dx}\left(\frac{1}{x}y\right) = 2 + \frac{1}{x}$ $\Leftrightarrow$ (1)

(1) $\Leftrightarrow \frac{y}{x} = 2x + \ln|x| + C \Leftrightarrow$ $\boxed{y = 2x^2 + x\ln|x| + Cx}$

[5] 3. Is the ODE below exact?

$\frac{\partial}{\partial s}\left(\frac{1}{r}+2s^2 r\right) = 4sr$ $\left(\frac{1}{r}+2s^2 r\right)dr + (2sr^2 - \cos(s))ds = 0$ $\frac{\partial}{\partial r}(2sr^2 - \cos(s)) = 4sr$

[6] 4. Suppose a brine containing 0.3 kilograms (kg) of salt per litre (L) runs into a tank initially filled with 400 L of water containing 2 kg of salt. The bring enters at 10 L/min, the mixture is kept uniform by stirring, and the mixture flows out at the same rate.

(a) Let $X(t)$ be the amount of salt in the tank at time $t$. Write down the ODE and initial conditions for $X(t)$.

$$\frac{dX(t)}{dt} = (0.3)10 - \frac{X(t)}{400}\cdot 10 \Leftrightarrow \frac{dX}{dt} = 3 - \frac{X}{40}, \quad X(0) = 2$$

(b) What is the mass of salt in the tank after 10 min?

$$\frac{dX}{dt} = \frac{120-X}{40} \Leftrightarrow \frac{dX}{120-X} = \frac{1}{40}dt \Rightarrow -\ln|120-X| = \frac{1}{40}t + H$$

$$\Rightarrow \ln|120-X| = -\frac{1}{40}t + C \Leftrightarrow |120-X| = e^{-\frac{1}{40}t}\cdot A$$

We have $X(0)=2$, so

$$|120-2| = e^0 \cdot A \Leftrightarrow A = 118$$

Also, $\because X(0) < 120$, we have $|120-X| = 120-X$ initially, so

$$120 - X = 118e^{-\frac{1}{40}t} \Leftrightarrow X = 120 - 118e^{-\frac{1}{40}t}$$

$$X(10) = 120 - 118e^{-\frac{1}{40}\cdot 10} = 120 - 118e^{-\frac{1}{4}}$$