

UBC ID #: _____ NAME (print): _____

Signature: Solutions



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THE UNIVERSITY OF BRITISH COLUMBIA

IRVING K. BARBER SCHOOL
OF ARTS AND SCIENCES
UBC OKANAGAN

Instructor: Rebecca Tyson Course: MATH 225
Date: Jan 31st, 2018 Time: 11:30am Duration: 35 minutes.
This exam has 4 questions for a total of 21 points.

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is a two-stage exam. You have 35 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 35 minutes.

Question:	1	2	3	4	Total
Points:	5	5	5	6	21
Score:					

Signature _____

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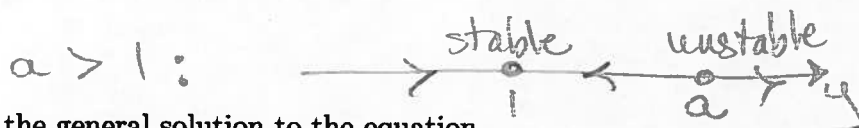
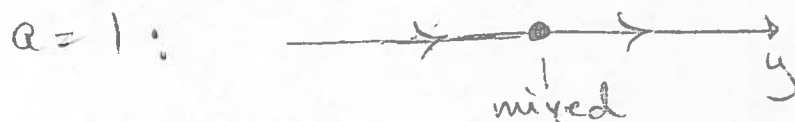
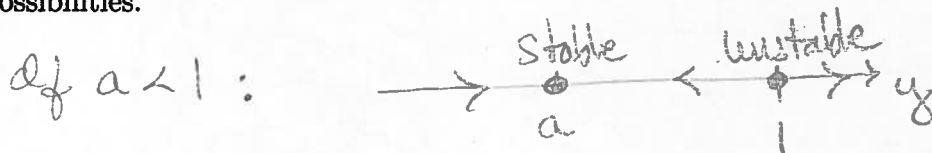


SPECIAL INSTRUCTIONS
 This exam has 7 questions for a total of 21 points.
 Each part is 30% of the total score.
 You have 45 minutes to complete the exam.

- Show and explain all your work unless the question directs otherwise. Significant figures are required.
- The use of a calculator is not permitted.
- Answer the question in the space provided on this question sheet. If you run out of room on an answer, ask for extra paper.
- This is a two-part exam. You have 45 minutes to complete the exam. You are responsible for the test and this exam group is responsible for the remaining 15 minutes.

Question	1	2	3	4	5	6	7	Total
Points	3	3	3	3	3	3	3	21
Score								

- 5] 1. Consider the ODE $y' = (y-1)(y-a)$. Sketch the phase line and state the nature of its steady states. Note that your answer depends on the value of a . You should consider all possibilities.



- 5] 2. Obtain the general solution to the equation

$$\frac{dy}{dx} = \frac{y}{x} + 2x + 1$$

$$\frac{dy}{dx} - \frac{1}{x}y = 2x + 1 \quad \therefore P(x) = -\frac{1}{x}, \quad Q(x) = 2x + 1$$

$$\mu(x) = e^{\int -\frac{1}{x} dx} = e^{-\ln|x|} = \frac{1}{|x|}$$

$$\text{take } \mu(x) = \frac{1}{x}$$

\therefore we have

$$\frac{1}{x} \frac{dy}{dx} - \frac{1}{x^2} y = 2 + \frac{1}{x} \quad \Leftrightarrow \quad \frac{d}{dx} \left(\frac{1}{x} y \right) = 2 + \frac{1}{x}$$

$$\int \frac{1}{x} \frac{dy}{dx} - \frac{1}{x^2} y = 2 + \frac{1}{x} \quad \Leftrightarrow \quad \frac{y}{x} = 2x + \ln|x| + C \quad \Leftrightarrow \quad \boxed{y = 2x^2 + x \ln|x| + Cx}$$

1. Consider the ODE $y' = (y-1)^2 - 4$. Sketch the phase line and state the nature of its steady states. State that your answer depends on the value of y . You do not need to justify your answer.

2. Obtain the general solution to the equation

$$y'' + 4y' + 4y = 1$$

- 5 3. Is the ODE below exact?

$$\frac{\partial}{\partial s} \left(\frac{1}{r} + 2s^2 \right) = 4sr$$

$$\left(\frac{1}{r} + 2s^2 \right) dr + (2sr^2 - \cos(s)) ds = 0$$

$$\frac{\partial}{\partial r} (2sr^2 - \cos(s)) = 4sr$$

- 6 4. Suppose a brine containing 0.3 kilograms (kg) of salt per litre (L) runs into a tank initially filled with 400 L of water containing 2 kg of salt. The brine enters at 10 L/min, the mixture is kept uniform by stirring, and the mixture flows out at the same rate.

- (a) Let $X(t)$ be the amount of salt in the tank at time t . Write down the ODE and initial conditions for $X(t)$.

$$\frac{dX(t)}{dt} = (0.3)10 - \frac{X(t)}{400} \cdot 10 \Leftrightarrow \frac{dX}{dt} = 3 - \frac{X}{40}, \quad X(0) = 2$$

- (b) What is the mass of salt in the tank after 10 min?

$$\begin{aligned} \frac{dX}{dt} &= \frac{120 - X}{40} \Leftrightarrow \frac{dX}{120 - X} = \frac{1}{40} dt \Leftrightarrow -\ln|120 - X| = \frac{1}{40}t + C \\ \Leftrightarrow \ln|120 - X| &= -\frac{1}{40}t + C \Leftrightarrow |120 - X| = e^{-\frac{1}{40}t} \cdot A \end{aligned}$$

We have $X(0) = 2$, so

$$|120 - 2| = e^0 \cdot A \Leftrightarrow A = 118$$

Also, $\because X(0) < 120$, we have $|120 - X| = 120 - X$ initially, so

$$120 - X = 118e^{-\frac{1}{40}t} \Leftrightarrow X = 120 - 118e^{-\frac{1}{40}t}$$

$$X(10) = 120 - 118e^{-\frac{1}{40} \cdot 10} = 120 - 118e^{-\frac{1}{4}}$$

(a) Is the ODE linear?

$$y'' + 2y' + 2y = \cos(x) + \sin(x)$$

(b) Suppose a curve contains 0.3 kilograms (kg) of salt for every 100 grams of water. The tank contains 10 kg of water. The mixture is kept uniform in the tank, and the mixture flows out of the tank at the same rate as it flows in. (a) For $Y(t)$ be the amount of salt in the tank at time t . Write down the ODE and initial conditions for $Y(t)$.

(b) What is the mass of salt in the tank after 10 min?