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a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

Irving K. Barber School
of Arts and Sciences
UBC Okanagan

Instructor: Rebecca Tyson Course: MATH 225
Date: Mar 14th, 2018 Time: 12:05pm Duration: 20 minutes.
This exam has 4 questions for a total of 32 points.

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is a two-stage exam. You have 35 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 35 minutes.

Question:	1	2	3	4	Total
Points:	4	5	8	15	32
Score:					

[4] 1. The differential equation $y'' + y = 0$ has the general solution $y(t) = c_1 \cos(t) + c_2 \sin(t)$. Determine the form of the particular solution for the differential equation below (DO NOT SOLVE!):

$$y'' + y = te^{3t}\cos(t) - 4\sin(t)$$

2. Consider the IVP

$$\frac{dy}{dx} = y(2 - x) + x^2, \qquad y(0) = 1.$$

[3] (a) Write out the ODE using the Backward Euler and Forward Euler formulae (do not solve for y_{n+1}).

[2] (b) Your friend chooses to obtain the solution using a different numerical method. After one step of size $h = 0.1$, the magnitude of the local error is ≈ 0.001. What can you say about the method your friend is using? How does it compare to the Backward Euler method?

3. Consider the mass-spring system with mass 2 kg, damping coefficient 1 kg/s, and spring constant 5/4 N/m. Let $x(t)$ represent the displacement of the mass as a function of time.

[2] (a) Write down the differential equation for $x(t)$ when the system is subject to the forcing $f(t) = \cos(3t/4)$.

[1] (b) Given that the solution to the homogeneous system is

$$x(t) = e^{-\frac{1}{4}t}\left(\cos\left(\frac{3}{4}t\right) + \sin\left(\frac{3}{4}t\right)\right),$$

what is the angular frequency of the homogeneous system?

(c) The general solution of the forced system is

$$x(t) = Ae^{(-b/2m)t}\sin\left(\frac{3}{4}t + \phi\right) + \frac{8}{\sqrt{37}}\sin\left(\frac{3}{4}t + \theta\right), \tag{1}$$

[2] i. Explain what the two terms in (1) represent.

[3] ii. What is the frequency gain of the forced system? How does it compare to the amplitude of the forcing itself? Explain.

15 4. Solve the initial value problem

$$w'' - 2w' + w = e^s \ln(s), \quad s > 0, \qquad w(1) = e, \quad w'(1) = -e.$$

(extra space for problem 4)