UBC ID #: ____________________ NAME (print): ____

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THE UNIVERSITY OF BRITISH COLUMBIA

IRVING K. BARBER SCHOOL
OF ARTS AND SCIENCES
UBC OKANAGAN

Instructor: Maria Martignoni Mseya Course: MATH 225
Date: Jan 30th, 2019 Time: 11:30am Duration: 35 minutes.
This exam has 5 questions for a total of 27 points.

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is a two-stage exam. You have 35 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 15 minutes.

Question:	1	2	3	4	5	Total
Points:	7	7	3	7	3	27
Score:						

7 1. Assume that the equation describing the growth population of a size N (with $N > 0$) over time t is given by the solutions of the ODE

$$N' = rN(1 - \frac{N}{K})(N - \mu) \tag{1}$$

Where μ and K are parameter values, with $\mu \ll K$.

(a) State the independent and the dependent variable of Eq. (1).

(b) Sketch the phase line of Eq. (1), and state the nature of the equilibria.

(c) What is the behaviour of the solutions for $t \to \infty$? Discuss all possible scenarios.

7 2. Solve the initial value problem

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^3 - 1 + \frac{y}{x}, \quad y(1) = 0$$

3 3. Match the direction field below with the correct differential equation. <u>SHOW YOUR WORK!</u> (in the absence of any explanation no marks will be given).

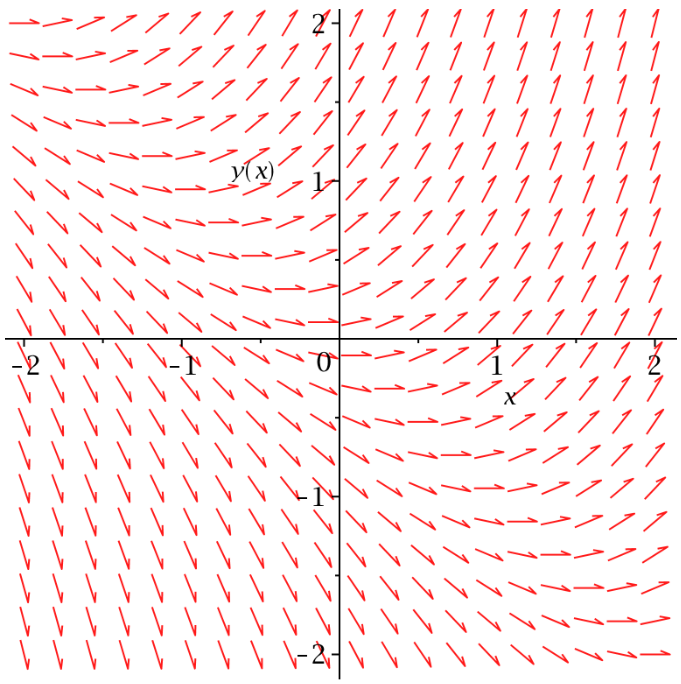
(a) $y' = -x/y$

(b) $y' = xy^2$

(c) $y' = x - y$

(d) $y' = x + y$

(e) $y' = x/y$



7 4. Consider the equation

$$-\,\mathrm{d}c = k\,c\;\mathrm{d}t, \quad c(0) = c_0 \tag{2}$$

where k is a positive constant.

(a) Solve Eq. (2)

(b) Assume that the variable c represents a concentration. Find an expression for the half-time, i.e. the time within which the concentration c decreases to 50% of the initial concentration.

[3] 5. Consider the differential equation

$$(2y^2 - \frac{y}{x}\sin x)\,\mathrm{d}x + (xy - \frac{y}{x}\cos y)\,\mathrm{d}y = 0\,. \tag{3}$$

Show that $\mu = \frac{x}{y}$ is an integrating factor of Eq. (3).