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THE UNIVERSITY OF BRITISH COLUMBIA

Irving K. Barber School
of Arts and Sciences
UBC Okanagan

Instructor: Maria Martignoni Mseya Course: MATH 225
Date: Mar 13th, 2019 Time: 11:30am Duration: 35 minutes.
This exam has 5 questions for a total of 25 points.

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is a two-stage exam. You have 35 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 15 minutes.

Question:	1	2	3	4	5	Total
Points:	4	6	5	6	4	25
Score:						

4 1. The motion of a swinging door is governed by

$$I\theta'' + b\theta' + k\theta = 0, \quad \theta(0) = \theta_0, \quad \theta'(0) = v_0$$

where θ is the angle that the door is open. I, b and k are positive constants indicating respectively the moment of inertia, damping constant and spring constant associated with the swinging door. θ_0 is the initial angle that the door is opened, and v_0 is the initial angular velocity imparted to the door

(a) Determine for which values of b the door will *not* continually swing back and forth when closing.

(b) Assume $I = 1$, $b = 0$, and $k = 1$. Write the down the form of the particular solution (do not solve for the coefficients) if the system described is subject to a forcing of:

- $F_0 \cos(t)$

- $F_0\, t\, \sin(t)$

[6] 2. Find a differential equation corresponding to the following general solutions

(a) $y(t) = c_1 e^{4t} + c_2 e^{-4t}$.

(b) $y(t) = c_1 t^4 + c_2 t^{-4}$

(c) $y(t) = c_1 e^{4t} + c_2 e^{-4t} + te^{4t}$

5 3. (a) Show that if $y_1(t)$ and $y_2(t)$ are linearly dependent on the interval I then their Wroskian is identically zero on I.

(b) Given that $y(x) = c_1 + c_2 x^2$ is a two-parameter family of solutions of $xy'' - y' = 0$ on the interval $(-\infty, \infty)$, show that constants c_1 and c_2 cannot be found so that a member of the family satisfies the initial conditions $y(0) = 0$, $y'(0) = 1$. Explain why this does not violate the existence and uniqueness theorem (the theorem is provided in the last page of the exam).

6 4. Find the general solution to

$$y'' - 6y' + 9y = t^{-3}e^{3t}$$

4 5. Consider the IVP

$$\frac{dy}{dx} = y - x\,, \quad y(0) = 1$$

(a) Write the forward Euler formula for the equation above.

(b) Show that $y_2 = 1 + 2h$.

Existence and Uniqueness Theorem: If p(t), q(t), and g(t) are continuous on an interval (a,b) that contains the point t_0, then for any choice of the initial values Y_0 and Y_1, there exists a unique solution $y(t)$ on the same interval (a, b) to the initial value problem

$$y''(t) + p(t)y'(t) + q(t)y(t) = g(t)\,, \quad y(t_0) = Y_0, \quad y'(t_0) = Y_1$$