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THE UNIVERSITY OF BRITISH COLUMBIA

IRVING K. BARBER SCHOOL
OF ARTS AND SCIENCES
UBC OKANAGAN

Instructor: Maria Martignoni Mseya Course: MATH 225
Date: Mar 13th, 2019 Time: 11:30am Duration: 35 minutes.
This exam has 5 questions for a total of 25 points.

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is a two-stage exam. You have 35 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 15 minutes.

Question:	1	2	3	4	5	Total
Points:	4	6	5	6	4	25
Score:						

[4] 1. The motion of a swinging door is governed by

$$I\theta'' + b\theta' + k\theta = 0, \quad \theta(0) = \theta_0, \quad \theta'(0) = v_0$$

where θ is the angle that the door is open. I, b and k are positive constants indicating respectively the moment of inertia, damping constant and spring constant associated with the swinging door. θ_0 is the initial angle that the door is opened, and v_0 is the initial angular velocity imparted to the door

(a) Determine for which values of b the door will *not* continually swing back and forth when closing.

Ch. equation $Ir^2 + br + K = 0$

$b^2 - 4IK \geq 0$

$b^2 \geq 4IK$

$b \geq \sqrt{4IK}$ $(b>0)$

(b) Assume $I = 1$, $b = 0$, and $k = 1$. Write ~~the~~ down the form of the particular solution (do not solve for the coefficients) if the system described is subject to a forcing of:

- $F_0 \cos(t)$

Hom. $\theta'' + \theta = 0$

Ch. eq. $r^2 + 1 = 0$

$r = \pm i \to y_h = c_1\cos(t) + c_2\sin(t)$

$y_p = t(A\cos(t) + B\sin(t))$

- $F_0\, t\, \sin(t)$

$y_p = t(At+B)(C\cos(t) + D\sin(t))$

$= t(A_1t + B_1)\cos(t) + t(A_2t + B_2)\sin(t)$

$AC = A_1$, $BC = B_1$

$AD = A_2$, $BD = B_2$

$\boxed{6}$ 2. Find a differential equation corresponding to the following general solutions

(a) $y(t) = c_1 e^{4t} + c_2 e^{-4t}$.

$r_1 = 4, \quad r_2 = -4$

Ch. equation $\quad (r-4)(r+4) = 0$

$r^2 - 16 = 0$

ODE: $\quad y'' - 16y = 0$

(b) $y(t) = c_1 t^4 + c_2 t^{-4}$

Cauchy-Euler

$r^2 - 16 = 0$

$a = 1, \quad b - a = 0, \quad c = -16$

$b = 1$

ODE: $\quad t^2 y'' + t y' - 16 y = 0$

(c) $y(t) = c_1 e^{4t} + c_2 e^{-4t} + te^{4t}$

Particular solution: $\quad te^{4t}, \quad y_h = c_1 e^{4t} + c_2 e^{-4t}$

ODE $\quad y'' - 16y = Ae^{4t}$

Find A:

$y_p = te^{4t}$

$y_p' = 4te^{4t} + e^{4t}$

$y_p'' = 4e^{4t} + 16te^{4t} + 4e^{4t}$

$= 16te^{4t} + 8e^{4t}$

$y_p'' - 16y = Ae^{4t}$

$\cancel{16te^{4t}} + 8e^{4t} - \cancel{16te^{4t}} = Ae^{4t}$

$A = 8$

ODE: $y'' - 16y = 8e^{4t}$

$\boxed{5}$ 3. (a) Show that if $y_1(t)$ and $y_2(t)$ are linearly dependent on the interval I then their Wroskian is identically zero on I.

If. lin. dep $y_1 = K y_2$

$$W = \begin{vmatrix} K y_2 & y_2 \\ K y_2' & y_2' \end{vmatrix} = K y_2 y_2' - K y_2' y_2 = 0 \quad \forall t$$

(b) Given that $y(x) = c_1 + c_2 x^2$ is a two-parameter family of solutions of $xy'' - y' = 0$ on the interval $(-\infty, \infty)$, show that constants c_1 and c_2 cannot be found so that a member of the family satisfies the initial conditions $y(0) = 0$, $y'(0) = 1$. Explain why this does not violate the existence and uniqueness theorem (the theorem is provided in the last page of the exam).

$y(0) = c_1 = 0$

$y'(x) = 2c_2 x$

$y'(0) = 0 = 1$ ↯ c_1, c_2 cannot be found such that $y(0) = 0$, $y'(0) = 1$

ODE: $xy'' - y' = 0$

Standard form:

Not continuous at $t = 0$

$$y'' - \left(\frac{1}{x}\right) y' = 0$$

A unique solution exists for $X_0 \in (0, \infty)$ or for $X_0 \in (-\infty, 0)$

A solution is not guarantee if $X_0 = 0$ (as in our case)

$\boxed{6}$ 4. Find the general solution to

$$y'' - 6y' + 9y = t^{-3}e^{3t}$$

Method of variation of parameter

Find y_1, y_2

$r^2 - 6r + 9 = 0$

$(r-3)^2 = 0$

$r = 3$ double root

$y_1 = e^{3t}$, $y_2 = te^{3t}$

$$\begin{cases} \cancel{e^{3t}} v_1' + t\cancel{e^{3t}} v_2' = 0 \\ 3\cancel{e^{3t}} v_1' + (\cancel{e^{3t}} + 3t\cancel{e^{3t}}) v_2' = t^{-3}\cancel{e^{3t}} \end{cases}$$

$$\Leftrightarrow \begin{cases} {}^{3\times} v_1' + {}^{3\times} t v_2' = 0 \\ 3v_1' + v_2' + 3t v_2' = t^{-3} \end{cases}$$

$0 - v_2' = -t^{-3}$

$v_2' = t^{-3}$

$v_2 = \int t^{-3}\,dt = -\frac{t^{-2}}{2}$

$v_1' = -t(t^{-3}) = -t^{-2}$

$v_1 = \int -t^{-2}\,dt = t^{-1}$

$y_{gen} = C_1 e^{3t} + C_2 t e^{3t} + \frac{1}{2t} e^{3t}$

$y_p = \frac{e^{3t}}{t} + \frac{te^{3t}}{2}(-t^{-2}) = \frac{e^{3t}}{t} - \frac{e^{3t}}{2t} = \frac{1}{2t} e^{3t}$

$\boxed{4}$ 5. Consider the IVP

$$\frac{dy}{dx} = y - x\,, \quad y(0) = 1$$

(a) Write the forward Euler formula for the equation above.

$$\frac{y_{n+1} - y_n}{h} = y_n - x_n$$

or

$$y_{n+1} = y_n + h(y_n - x_n)$$

(b) Show that $y_2 = 1 + 2h$.

$$y_1 = \overset{y_0}{1} + h(1 - \overset{x_0}{0}) = 1 + h$$

$$y_2 = \underbrace{(1+h)}_{y_1} + h(\underbrace{1 + \cancel{h}}_{y_1} - \overset{x_1}{\cancel{h}}) = 1 + 2h$$

Existence and Uniqueness Theorem: If p(t), q(t), and g(t) are continuous on an interval (a,b) that contains the point t_0, then for any choice of the initial values Y_0 and Y_1, there exists a unique solution $y(t)$ on the same interval (a, b) to the initial value problem

$$y''(t) + p(t)y'(t) + q(t)y(t) = g(t)\,, \quad y(t_0) = Y_0, \quad y'(t_0) = Y_1$$