UBC ID #: _____ NAME (print): _____

Signature: _____

a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

Irving K. Barber School
of Arts and Sciences
UBC Okanagan

Instructor: Rebecca Tyson Course: MATH 225
Date: Feb 9th, 2022 Time: 4:00pm Duration: 35 minutes.
This exam has 4 questions for a total of 28 points.

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. **Answers without accompanying work are worth zero.** Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is a two-stage exam. You have 35 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 35 minutes.

1. Consider the ODE

$$\frac{dy}{dt} = f(y) = y\cos(y) + a, \tag{1}$$

where a is a constant. The function $f(y)$ for the case $a = 0$ is plotted in Figure 1.

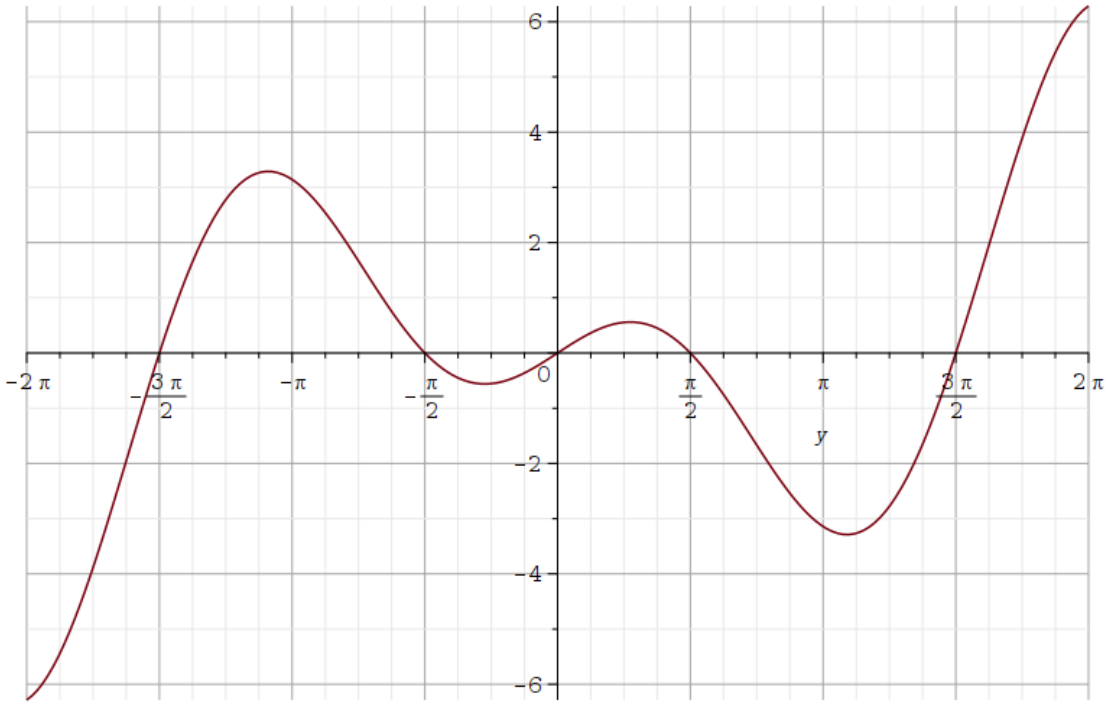


Figure 1: Plot of the function $f(y)$ defined in (1).

[1] (a) The ODE (1) is autonomous. Why?

[3] (b) In the space below, sketch the phase line between $\pm 2\pi$ for the case $a = 0$, and indicate the stability of each steady state.

3 (c) How would the stability of the steady states change if $a = 2$ or $a = -2$? Explain.

7 2. Solve the ODE

$$(\sin(x) + \ln(y))\,dx + \left(\frac{x}{y} + e^y\right)dy = 0.$$

3. Consider the initial value problem

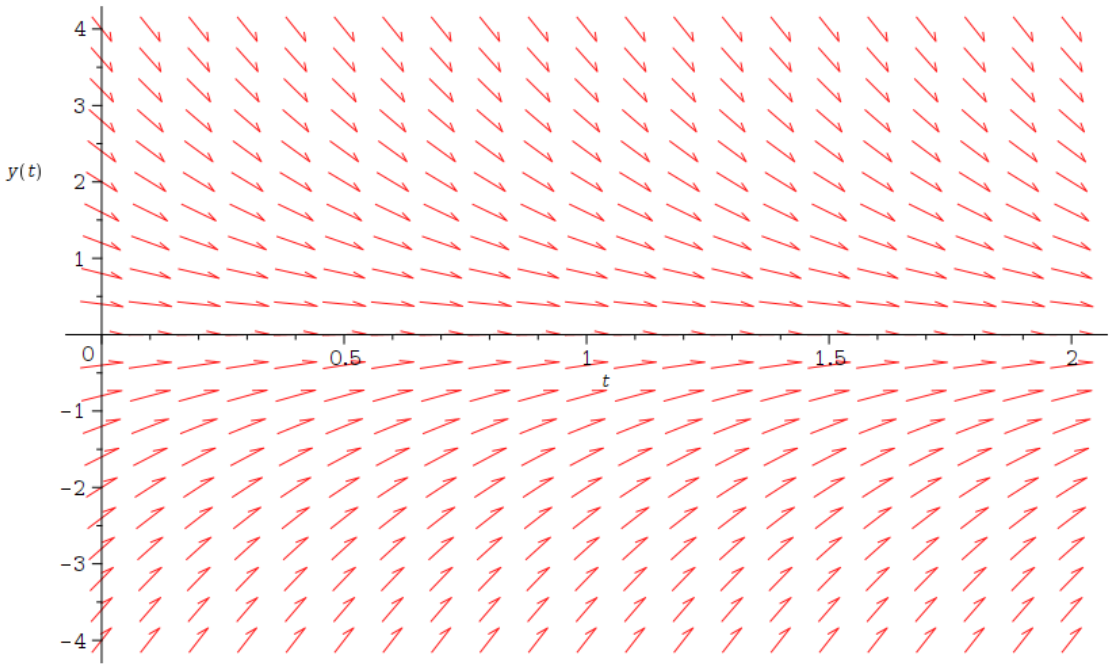
$$\frac{dy}{dt} = te^{-2t} - 2y, \quad y(0) = y_0.$$

5 (a) The ODE is linear. Solve it.

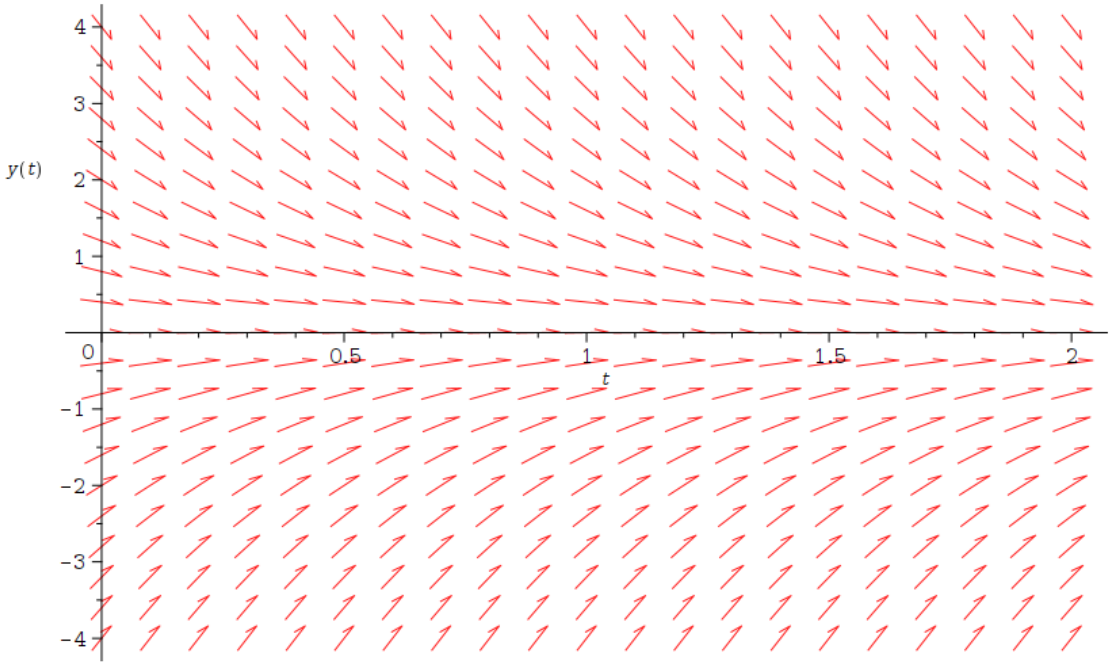
2 (b) How does the value of y_0 affect $\lim_{t\to\infty} y(x)$? Explain.

4. Numerical methods.

2 (a) On the direction field below, starting at the point $(t, y) = (0.5, 3)$ and using $h = 1$, carefully plot one step of the Forward Euler method, including the relevant slope arrows. Add explanatory text if necessary.



4 (b) On the direction field below, starting at the point $(t, y) = (0.5, 3)$ and using $h = 1$, carefully plot one step of the Heun method, including the relevant slope arrows. Add explanatory text if necessary.



1 (c) Why is the Heun method more accurate?

Question:	1	2	3	4	Total
Points:	7	7	7	7	28
Score:					