UBC ID #: ____________________ NAME (print): ____

Signature: ____

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THE UNIVERSITY OF BRITISH COLUMBIA

Irving K. Barber School
of Arts and Sciences
UBC Okanagan

Instructor: Rebecca Tyson Course: MATH 225
Date: Feb 7th, 2022 Time: 4:00pm Duration: 35 minutes.
This exam has 4 questions for a total of 25 points.

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. **Answers without accompanying work are worth zero.** Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is a two-stage exam. You have 35 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 35 minutes.

1. The direction field for

$$\frac{dy}{dx} = 2x + y \tag{1}$$

is shown in Figure 1. Use it to answer the questions below.

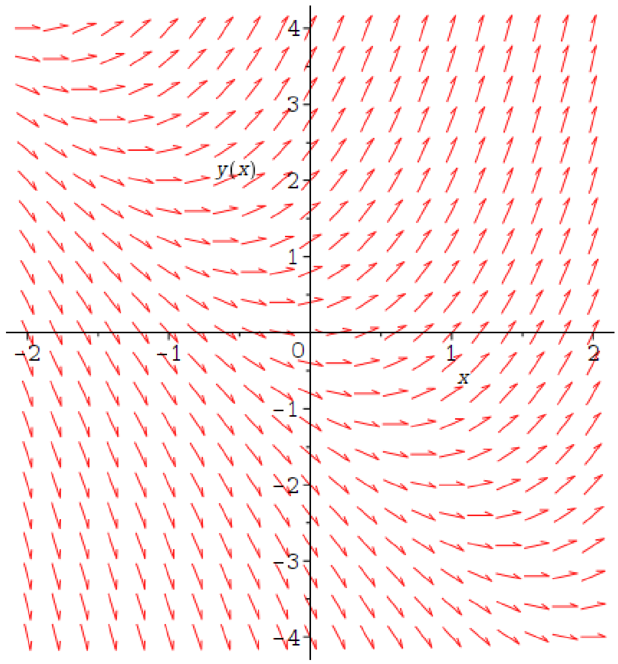

Figure 1: Figure for problem 1.

2 (a) Sketch the solution curve that passes through the point (-2,3). Label the curve $y_1(x)$.

1 (b) What can you say about $y_1(x)$ as $x \to \infty$?

2 (c) Sketch the solution curve that passes through the point (0,-2) and label it $L(x)$.

1 (d) What can you say about the solution $y_1(x)$ as $x \to -\infty$? Be specific.

4 2. Solve the initial value problem

$$e^x \frac{dy}{dx} = y^2, \quad y(0) = 1,$$

using separation of variables. Show all your work.

3. Consider the ODE

$$(3y^2 - t^2)dy - 2tydt = 0. \tag{2}$$

2 (a) Show that (2) is exact.

7 (b) Solve the ODE (2). You should obtain an implicit solution for $y(t)$.

4. . Consider the ODE

$$\frac{dN}{dt} = N(1 - N).$$

2 (a) Write the Forward Euler (FE) and Backward Euler (BE) approximations (in the BE case, do not solve for N_{t+1}).

2 (b) The direction field for the ODE and the true solution when $N(0) = 0.1$ is shown in Figure 2. Show two steps of the FE and BE methods on the direction field using $h = 2$.

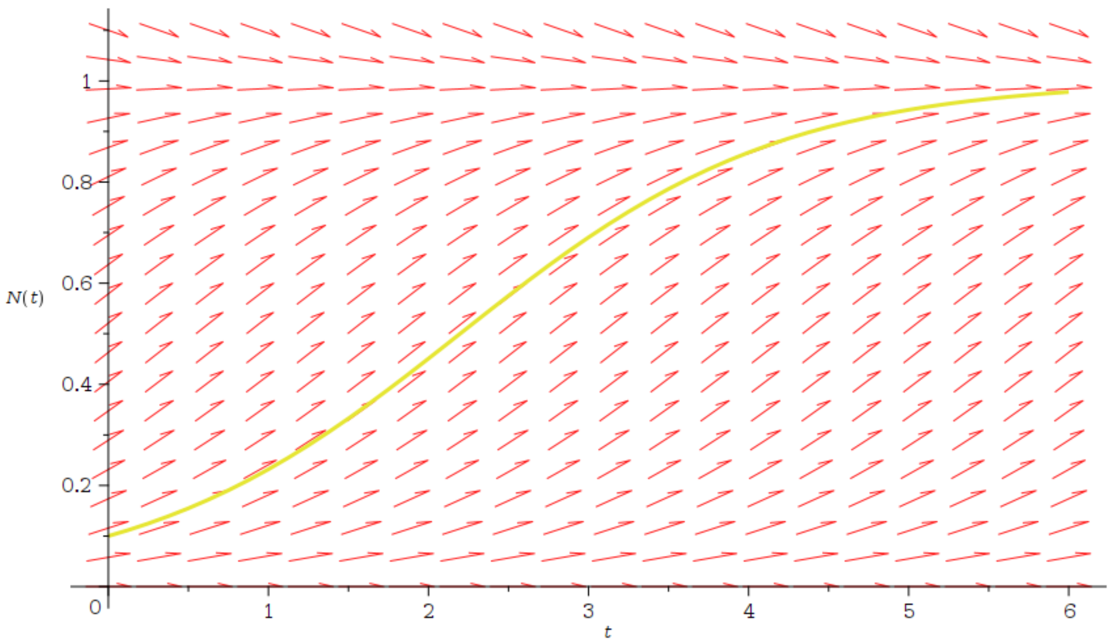

Figure 2: Figure for problem 4.

2 (c) Why does the FE method initially underestimate the true solution? Explain.