UBC ID #: ____________ NAME (print): ____________________

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a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

Irving K. Barber School
of Arts and Sciences
UBC Okanagan

Instructor: Rebecca Tyson Course: MATH 225
Date: Mar 23rd, 2022 Time: 4:00pm Duration: 35 minutes.
This exam has 5 questions for a total of 48 points.

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. **Answers without accompanying work are worth zero.** Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is a two-stage exam. You have 35 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 35 minutes.

1. The homogeneous ODE

$$y'' + 5y' + 6y = 0,$$

has general solution

$$y(t) = c_1 e^{-3t} + c_2 e^{-2t}.$$

2 (a) Write the FORM of the particular solution for the ODE $y'' + 5y' + 6y = 3t^2 + 2e^t$.

2 (b) Write the FORM of the particular solution for the ODE $y''+5y'+6y = 5e^{-3t}+2te^{-2t}$.

4 2. What are the resonance frequency and maximum possible frequency gain for the mass-spring system with mass 4, spring constant 1, and damping coefficient 2 (assume all numbers are given in a consistent system of units)? Your answer should be exact.

3. Consider the differential equation $\frac{1}{2}y'' + 2y = \tan(2t)$, which has fundamental solution set $\{\cos(2t), \sin(2t)\}$.

8 (a) Find a particular solution. *This integral may be helpful:* $\int(\tan(2t)\sin(2t)dt) = \frac{1}{2}(-\sin(2t) + \ln(\sec(2t) + \tan(2t)))$

2 (b) Write the general solution.

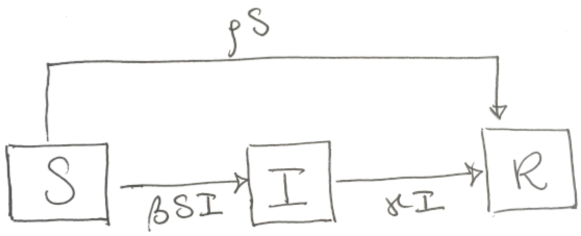
4. A 3 kg mass is attached to a spring with stiffness $k = 75$ N/m. The mass is displaced 1 m to the right and given a velocity of 5 m/sec to the left. The damping force is negligible.

6 (a) Find the equation of motion of the mass.

7 (b) Write the solution as a single phase-shifted sine, and sketch it. Include labels for the amplitude, phase shift, and period. The phase shift should be exact and in radians.

3 (c) How long after release does the mass pass through the equilibrium position? Include appropriate units.

5. Consider a disease for which immunity does not wane and for which there is a vaccine. We can draw the compartmental diagram for this disease as



Note that the total population N is constant, so $S + I + R = N$, and R includes people who are removed either by getting vaccinated, recovering from the disease, or dying from the disease. The parameter ρ is the vaccination rate.

3 (a) Write down the system of ODEs for this model.

7 (b) If we let $u = S/N$, $v = I/N$, $w = R/N$, and $\tau = \gamma\, t$, then we obtain the following two ODEs for u and v:

$$\frac{du}{d\tau} = \frac{-\beta N}{\gamma} uv - \frac{\rho}{\gamma} u, \qquad \frac{dv}{d\tau} = \frac{\beta N}{\gamma} uv - v.$$

Compute the Jacobian at the disease-free equilibrium (1,0). Under what conditions will an epidemic occur?

4 (c) Figure 1 shows (top plot) $I(t)$ for different values of ρ, and (bottom plot) peak and final size as functions of the vaccination rate ρ, for $R_0 = 3.0$ and $\gamma = 1$. Explain the results, i.e., explain why vaccination has the observed effect on the epidemic curve, peak size, and final size.

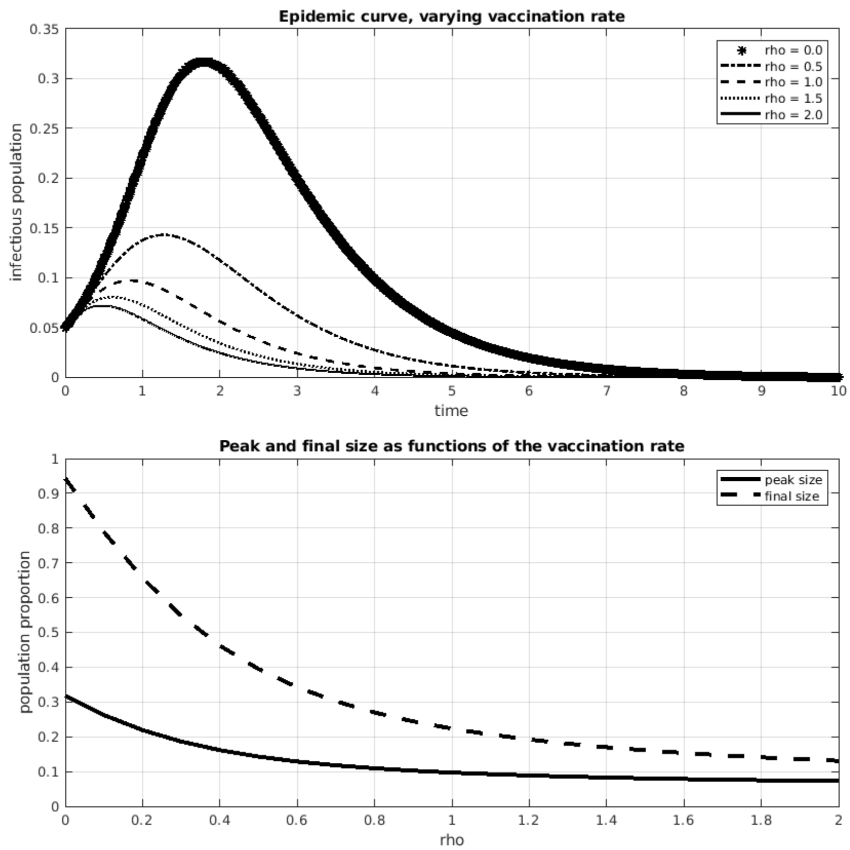


Figure 1: Output of the model.

Question:	1	2	3	4	5	Total
Points:	4	4	10	16	14	48
Score:						