UBC ID #: ____________________ NAME (print): ______________________________

Signature: ____

a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

Irving K. Barber School
of Arts and Sciences
UBC Okanagan

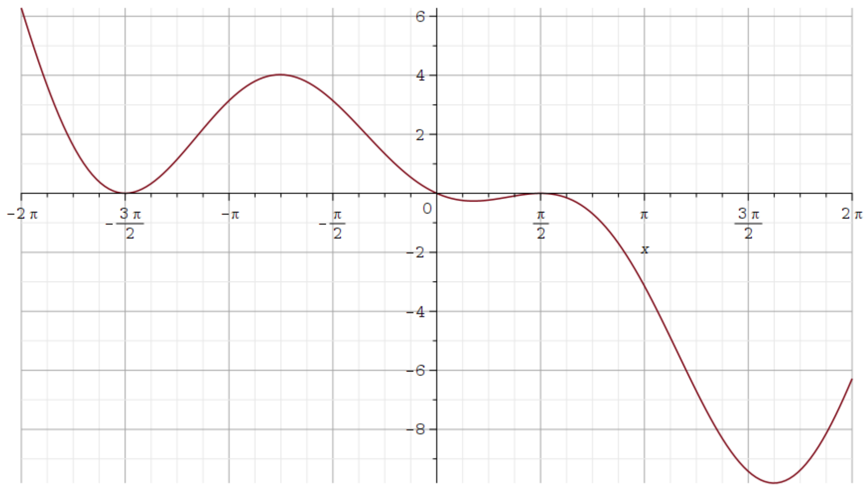
Instructor: Rebecca Tyson Course: MATH 225
Date: Feb 6th, 2023 Time: 4:00pm Duration: 35 minutes.
This exam has 6 questions for a total of 24 points.

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. **Answers without accompanying work are worth zero.** Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is a two-stage exam. You have 35 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 35 minutes.

1. The figure below is a plot of $f(x)$. Assume that outside the interval shown, the function never again crosses the horizontal axis. More specifically, the function is continuously increasing for $x < -3\pi/2$, peaks shortly to the right of $x = 2\pi$ and is continuously decreasing thereafter (for $x > 2\pi$).



3 (a) Use the horizontal axis (i.e., the line $f(x) = 0$) as your phase axis, and sketch the phase line for the ODE $x' = f(x)$. State the nature of the equilibria.

(b) Now imagine shifting the function $f(x)$ up or down by an arbitrary amount a.

2 i. What is the smallest shift size a at which the phase line has exactly two steady states? Specify if the shift is up or down.

2 ii. What are the two steady states and what is their stability? *Hint: You might find it useful to draw a horizontal line through the plot above, in the appropriate place, and indicate the steady states on that new line.*

[4] 2. Solve the ODE

$$\frac{dy}{dx} + xy^2 = 0$$

Make sure you give *all* of the solutions!

[5] 3. Solve the ODE

$$x\frac{dy}{dx} + 3(y + x^2) = 1$$

2 4. Find the most general function $R(p,q)$ so that the equation below is exact.

$$R(p,q)dq + (q\cos(p) + e^q)dp = 0$$

2 5. Set up the partial fraction decomposition (i.e. just set up the fractions - do not solve for the coefficients!) of

$$\frac{1}{1-x^4} = \frac{1}{(1-x)(1+x)(1+x^2)}.$$

6. Numerical solution of the ODE for $r(t)$ (not shown), using some unknown method, yields the results shown below.

stepsize	function value	difference
$h = 0.1$	$r(2) = 2.28835$	
$h = 0.05$	$r(2) = 2.26262$	
$h = 0.025$	$r(2) = 2.24945$	
$h = 0.0125$	$r(2) = 2.24279$	
$h = 0.00625$	$r(2) = 2.23943$	
$h = 0.003125$	$r(2) = 2.23775$	
$h = 0.0015625$	$r(2) = 2.23691$	

2 (a) Why does the value of $r(2)$ keep changing?

2 (b) Based on the information given, determine the value of $r(2)$ within two decimal places (± 0.01). Fill in the table as you do this, and explain how you arrived at your answer.

Question:	1	2	3	4	5	6	Total
Points:	7	4	5	2	2	4	24
Score:							