UBC ID #: ____________ NAME (print): ______________________

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a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

Irving K. Barber School
of Arts and Sciences
UBC Okanagan

Instructor: Rebecca Tyson Course: MATH 225
Date: Feb 12th, 2024 Time: 8:00am Duration: 35 minutes.
This exam has 5 questions for a total of 32 points.

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. **Answers without accompanying work are worth zero.** Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is a two-stage exam. You have 35 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 35 minutes.

1. The growth of a certain population $N(t)$ is described by

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{3}\right)(N-1), \tag{1}$$

where $r > 0$ and $t \geq 0$.

$\boxed{5}$ (a) Sketch the phase line. Label the steady states with their stability.

$\boxed{2}$ (b) Suppose $N(0) > 0$. What is $\lim_{t\to\infty} x(t)$?

$\boxed{6}$ 2. Solve the IVP

$$y'' + 2y' + y = 0, \qquad y(0) = 0, \quad y'(0) = 3.$$

$\boxed{5}$ 3. Consider the IVP

$$t\frac{dy}{dt} - y = t^2\sin(t), \qquad y(\pi) = 0, \quad t \geq 0.$$

The ODE is linear in y. Use this information to solve the IVP.

$\boxed{6}$ 4. Verify that the ODE below is exact, and solve it.

$$(e^{x+y} + 2x)dx + (e^{x+y} - 2y)dy = 0$$

5. Consider the ODE:

$$\frac{dx}{dt} = \frac{1}{x}. \tag{2}$$

$\boxed{2}$ (a) Write down the formula for the Forward Euler approximation with stepsize h.

$\boxed{3}$ (b) Write down the formula for the Heun approximation with stepsize h.

$\boxed{3}$ (c) The Taylor Series expansion of $x(t_n + h)$ around $x(t_n) = x_n$ is

$$x(t_n + h) = x_n + h\, \frac{dx}{dt}\bigg|_{x_n} + \frac{h^2}{2}\, \frac{d^2x}{dt}\bigg|_{x_n} + O(h^3).$$

Rewrite the derivatives (dx/dt and d^2x/dt^2) in the expansion in terms of $f(x) = 1/x$, and then compare the expansion with the **Forward Euler** formula you gave in part (a). How are they similar? What is the meaning of the difference?

Question:	1	2	3	4	5	Total
Points:	7	6	5	6	8	32
Score:						