UBC ID #: ____________ NAME (print): ____________________

Signature: ________________________________

a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

Irving K. Barber School
of Arts and Sciences
UBC Okanagan

Instructor: Rebecca Tyson Course: MATH 225
Date: Mar 25th, 2024 Time: 8:00am Duration: 35 minutes.
This exam has 5 questions for a total of 37 points.

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. **Answers without accompanying work are worth zero.** Simplify all answers.
- The use of a calculator is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is a two-stage exam. You have 35 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 35 minutes.

$\boxed{2}$ 1. Consider the mass-spring system

$$my'' + by' + ky = 0. \tag{1}$$

Assume that all of the parameters are non-negative. Under what conditions is the solution of (1) considered to be underdamped?

$\boxed{3}$ 2. Consider the initial value problem

$$y'' + 0.1y' + 25y = 2\cos(\gamma t), \qquad y(0) = 1, \quad y'(0) = 0. \tag{2}$$

For which integer value of γ will the particular solution have the largest magnitude? What is this frequency called?

3. Consider the equation

$$y'' + 25y = \cos(5t). \tag{3}$$

5 (a) Use the method of undetermined coefficients to find a particular solution.

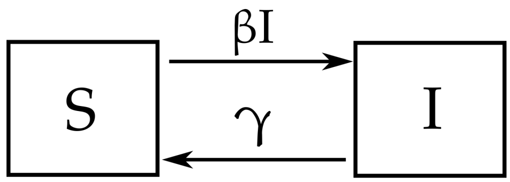
3 (b) Sketch the particular solution and give the quasiperiod.

$\boxed{8}$ 4. Use the method of variation of parameters to find a general solution to the differential equation

$$y'' + 2y' + y = e^{-t}. \tag{4}$$

Be sure to work from the system of two constraints on $v_1'(t)$ and $v_2'(t)$.

5. Consider the SIS disease diagram below. Assume also that $S + I = N$, a constant.



$\boxed{2}$ (a) Write the ODEs for this model.

$\boxed{4}$ (b) Non-dimensionalise the model using $u = S/N$, $v = I/N$, and $\tau = \gamma t$. Identify R_0.

$\boxed{1}$ (c) Write the population constraint in terms of u and v.

$\boxed{9}$ (d) In the phase plane, sketch the nullclines and locate the steady states in the case $R_0 > 1$ (use the dimensionless equations). Give their coordinates. What is each steady state called?

Question:	1	2	3	4	5	Total
Points:	2	3	8	8	16	37
Score:						