Math 319 - Differential Equations II
Pre-Reading Assignment # 4
due 10am Tue Sep 23rd, via email

Reading pages 578-580 (first three pages of section 10.3) in your text.

Questions Answer the question below to the best of your ability. It is a very short question, and shouldn't take you more than about half a page to complete.

1. In Example 2, the text claims that the three integrals,

$$\int_{-L}^{L} \sin\left(\frac{n\pi x}{L}\right) \cos\left(\frac{n\pi x}{L}\right) dx, \tag{1}$$

$$\int_{-L}^{L} \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{n\pi x}{L}\right) dx, \tag{2}$$

$$\int_{-L}^{L} \cos\left(\frac{n\pi x}{L}\right) \cos\left(\frac{n\pi x}{L}\right) dx, \tag{3}$$

all evaluate to 0 when $m \neq n$, and to L when $m = n \neq 0$. They use trigonometric identities and a symmetry argument to arrive at this result.

   (a) Confirm the result above for $m \neq n$ by actually computing the first integral (this is the one that we started in class).

   (b) Use the plots in Figure 10.5 to confirm the result above for the integrals (2) in the following way. Determine which curves in Figure 10.5 are relevant to the integrals in (2) (consider both the $m \neq n$ and $m = n \neq 0$ cases. Sketch these integrals in your homework, and fill in the relevant areas, showing which ones are negative and which ones are positive. Deduce from your sketch what the integral should be in the two cases.

   (c) Repeat for the integrals in (3), considering the three cases $m \neq n$, $m = n \neq 0$, and $m = n = 0$ cases.

2. What is the x-dependent BVP (boundary value problem) that arises when you apply the separation of variables technique to the PDE problem

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \tag{4}$$

with boundary conditions

$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(L,t) = 0, \text{ for } t > 0, \tag{5}$$

and initial condition

$$u(x,0) = f(x), \text{ for } 0 \leq x \leq L? \tag{6}$$