



a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

IRVING K. BARBER SCHOOL
 OF ARTS AND SCIENCES
 UBC OKANAGAN

Instructor: Rebecca Tyson Course: MATH 319
 Date: Oct 2nd, 2014 Time: 12:30pm Duration: 45 minutes.
 This exam has 4 questions for a total of 20 points.

NAME: _____ NAME: _____

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SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

This is a two-stage exam. You have 45 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 35 minutes.

Problem Number	1	2	3	4	Total
Points Earned					
Points Out Of	5	5	6	4	20

- 5 1. Use the method of characteristics to transform the PDE below into an ODE. Sketch the characteristic lines. *Note: Do not solve the ODE!!*

$$2u_x - 3u_y + x^2u = \frac{1}{y} \quad (1)$$

- 5 2. Use the technique of “separation of variables” to transform the PDE below into a pair of ODEs. At a critical point in the calculations, why can you set both sides equal to $-\lambda$? (*Note: Simply state the ODEs; do not solve them!*)

$$\frac{\partial^2 u}{\partial t^2} - k \frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2} + \mu e^{ax} u \quad (2)$$

Note that k , μ , and a are constants.

- 6 3. What are the eigenfunctions and eigenvalues of the BVP below? Assume $\rho \in \mathbb{R}$, $\rho > 0$.
Note: This assumption gives you just one case to consider.

$$F''(x) + 2F'(x) + (1 + \rho^2)F(x) = 0, \quad 0 < x < 2\pi, \quad (3a)$$

$$F(0) = F(2\pi) = 0. \quad (3b)$$

4. Consider the function

$$f(x) = 1 - e^x, \quad 0 < x < 2. \quad (4)$$

A sketch of the function is shown in Figure 1 (last page of the test).

- 2 (a) On the interval $[-6, 6]$, sketch the function to which the Fourier sine approximation of $f(x)$ converges.

- 2 (b) The Fourier sine series of $f(x)$ is written

$$f(x) = \sum_{n=1}^{\infty} b_n \sin(\gamma x).$$

- (a) What is γ ?

- (b) Write the formula for the coefficients b_n . For Bonus Points: solve for the coefficients!

Figures and A Few Integrals

Some integrals you may find useful:

$$\int x \sin(\rho x) dx = -\frac{x}{\rho} \cos(\rho x) + \frac{1}{\rho^2} \sin(\rho x) \quad (5)$$

$$\int x \cos(\rho x) dx = \frac{x}{\rho} \sin(\rho x) + \frac{1}{\rho^2} \cos(\rho x) \quad (6)$$

$$\int e^x \sin(\rho x) dx = \frac{e^x}{\rho^2 + 1} [\sin(\rho x) - \rho \cos(\rho x)] \quad (7)$$

$$\int e^x \cos(\rho x) dx = \frac{e^x}{\rho^2 + 1} [\rho \sin(\rho x) + \cos(\rho x)] \quad (8)$$

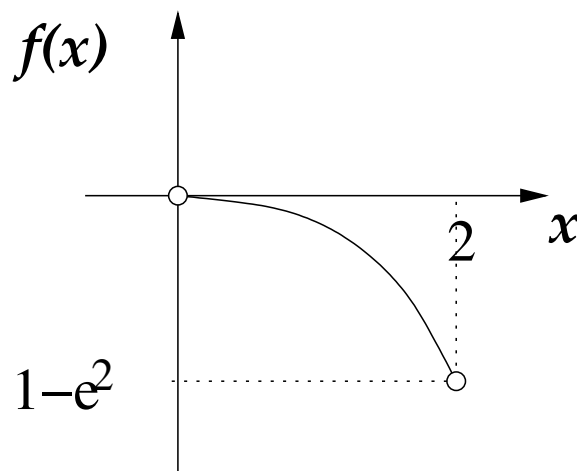


Figure 1: Plot of $f(x)$ for question 4.