a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

Irving K. Barber School
of Arts and Sciences
UBC Okanagan

Instructor: Rebecca Tyson Course: MATH 319
Date: Oct 2nd, 2014 Time: 12:30pm Duration: 45 minutes.
This exam has 4 questions for a total of 20 points.

NAME: ____________________

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

This is a two-stage exam. You have 45 minutes to complete the exam individually, then you will hand in the tests and join your group to redo the test as a group in the remaining 35 minutes.

Problem Number	1	2	3	4	**Total**
Points Earned					
Points Out Of	5	5	6	4	20

$\boxed{5}$ 1. Use the method of characteristics to transform the PDE below into an ODE. Sketch the characteristic lines. *Note: Do not solve the ODE!!*

$$2u_x - 3u_y + x^2 u = \frac{1}{y} \tag{1}$$

$\boxed{5}$ 2. Use the technique of "separation of variables" to transform the PDE below into a pair of ODEs. At a critical point in the calculations, why can you set both sides equal to $-\lambda$? *(Note: Simply state the ODEs; do not solve them!)*

$$\frac{\partial^2 u}{\partial t^2} - k\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2} + \mu e^{ax} u \tag{2}$$

Note that k, μ, and a are constants.

$\boxed{6}$ 3. What are the eigenfunctions and eigenvalues of the BVP below? Assume $\rho \in \mathbb{R}$, $\rho > 0$.
Note: This assumption gives you just one case to consider.

$$F''(x) + 2F'(x) + (1+\rho^2)F(x) = 0, \qquad 0 < x < 2\pi, \tag{3a}$$

$$F(0) = F(2\pi) = 0. \tag{3b}$$

4. Consider the function

$$f(x) = 1 - e^x, \qquad 0 < x < 2. \tag{4}$$

A sketch of the function is shown in Figure 1 (last page of the test).

2 (a) On the interval $[-6, 6]$, sketch the function to which the Fourier sine approximation of $f(x)$ converges.

2 (b) The Fourier sine series of $f(x)$ is written

$$f(x) = \sum_{n=1}^{\infty} b_n \sin(\gamma x).$$

(a) What is γ?

(b) Write the formula for the coefficients b_n. *Note: Do not solve for the coefficients!*

Figures and A Few Integrals

Some integrals you may find useful:

$$\int x\sin(\rho x)\,dx = -\frac{x}{\rho}\cos(\rho x) + \frac{1}{\rho^2}\sin(\rho x) \tag{5}$$

$$\int x\cos(\rho x)\,dx = \frac{x}{\rho}\sin(\rho x) + \frac{1}{\rho^2}\cos(\rho x) \tag{6}$$

$$\int e^x\sin(\rho x)\,dx = \frac{e^x}{\rho^2+1}\left[\sin(\rho x) - \rho\cos(\rho x)\right] \tag{7}$$

$$\int e^x\cos(\rho x)\,dx = \frac{e^x}{\rho^2+1}\left[\rho\sin(\rho x) + \cos(\rho x)\right] \tag{8}$$

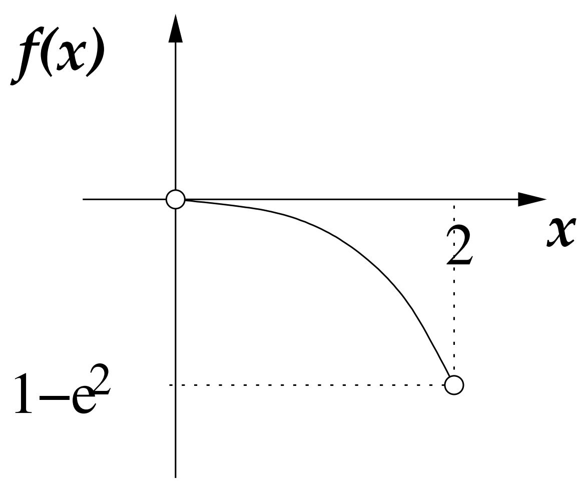

Figure 1: Plot of $f(x)$ for question 4.