Math 319 - Differential Equations II
Assignment # 1

Instructions: You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

Note: The first three problems below are review problems lifted directly from Math 225.

1. Find the general solution of the given second-order differential equation:
   (a) $4y'' + y' = 0$
   (b) $y'' - y' - 6y = 0$
   (c) $12y'' - 5y' - 2y = 0$

2. Solve the initial value problem
$$y'' + y' + 2y = 0, \qquad y(0) = 1, \quad y'(0) = 0$$

3. Consider the differential equation $y'' + \lambda\ y = 0$, with boundary conditions $y(0) = 0$, and $y(\pi/2) = 0$. This is called a *boundary value problem*, because instead of conditions on y and y' at a single time (say 0 *or* $\pi/2$), they are given both on y and at different times (0 *and* $\pi/2$). Is it possible to determine values of λ so that the problem possesses
   (a) only trivial solutions?
   (b) some nontrivial solutions?

4. Determine and graph the family of characteristic lines for the PDE $u_x(x,t) + u_t(x,t) = 0$.

5. Find the general solution of $u_x + u_y + u = e^{-3y}$.

6. Consider the hyperbolic PDE (also called the "wave equation" or "vibrating string equation")
$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}, \tag{1}$$
with boundary conditions and initial values given by
$$u(0,t) = u(4,t) = 0, \quad \text{and} \quad u(x,0) = 4\sin\left(\frac{3\pi x}{4}\right). \tag{2}$$
   (a) Show that
$$u(x,t) = K\sin\left(\frac{3\pi x}{4}\right)\cos\left(\frac{6\pi t}{4}\right) \tag{3}$$
   is a solution to (1) with (2). What must be the value of K?
   (b) Sketch the solution at $t = 0$ and at $t = 0.5$.

7. By inspection, determine the coefficients b_n on the left hand side
$$\sum_{n=1}^{\infty} b_n \sin(n\pi x) = 5\sin(2\pi x) - 7\sin(5\pi x) + 2\sin(9\pi x).$$
The function on the left hand side is called a "Fourier Series." *Hint: The functions* $\sin(n\pi x)$ *are all linearly independent.*