Math 319 - Differential Equations II
Assignment # 2
due Mon Oct 3rd, 11:30am, SCI 386

Instructions: You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

The assignment may be done with up to 4 other classmates (i.e. total group size: no more than 5). If you collaborate with classmates, the group should hand in one document with all contributing names at the top.

1. Use the method of separation of variables to derive the set of ODEs associated with the following partial differential equation for $u = u(x,t)$:

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + u = \alpha^2 \frac{\partial^2 u}{\partial x^2}.$$

2. Use separation of variables to solve the heat flow problem

$$\begin{aligned} &\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}, && 0 < x < \pi, \quad t > 0, \\ &u(0,t) = u(\pi,t) = 0, && t > 0, \\ &u(x,0) = \sin(x) - 7\sin(3x) + \sin(5x), && 0 < x < \pi. \end{aligned}$$

3. The Legendre polynomials, $P_n(x)$, are orthogonal on the interval $[-1,1]$ with respect to the weight function $w(x) = 1$. Use the fact that

$$P_0(x) = 1, \quad P_1(x) = x, \quad \text{and} \quad P_2(x) = \frac{3}{2}x^2 - \frac{1}{2}$$

   to find the first three coefficients in the expansion $f(x) = a_0P_0(x) + a_1P_1(x) + a_2P_2(x) + ...$, where $f(x)$ is the function given by

$$f(x) = \begin{cases} -1, & -1 < x < 0, \\ 1, & 0 < x < 1. \end{cases}$$

4. Obtain (a) the Fourier sine series, and (b) the Fourier cosine series for the function

$$f(x) = x + \pi \quad \text{on} \quad 0 < x < \pi.$$

   For each series, discuss the convergence and show graphically (using Maple) the function represented by the series for all x (your graph should show three periods of the function).

5. Consider the function

$$f(x) = \begin{cases} |x|, & \text{on} \quad -\pi \le x \le \pi \\ f(x+2\pi), & \text{else.} \end{cases}$$

   Use Maple to investigate the speed with which the Fourier series for f converges. In particular, determine how many terms are needed so that the error is no greater than 0.05 for all x in the interval $[-1,1]$. Your solution should include plots of $f(x)$ and the truncated Fourier series $s_m(x)$, as well as plots of the error $e_m(x) = |f(x) - s_m(x)|$ for various values of m.