Assisgnment #2, Problem #5, Maple portion

First I define the periodic function *f(x),* and its Fourier series.

> $f := x \rightarrow piecewise(-\text{Pi} < x \textbf{ and } x < \text{Pi}, |x|, -3\cdot\text{Pi} < x \textbf{ and } x < -\text{Pi}, |x + 2\cdot\text{Pi}|, \text{Pi} < x \textbf{ and } x < 3\cdot\text{Pi}, |x - 2\cdot\text{Pi}|);$

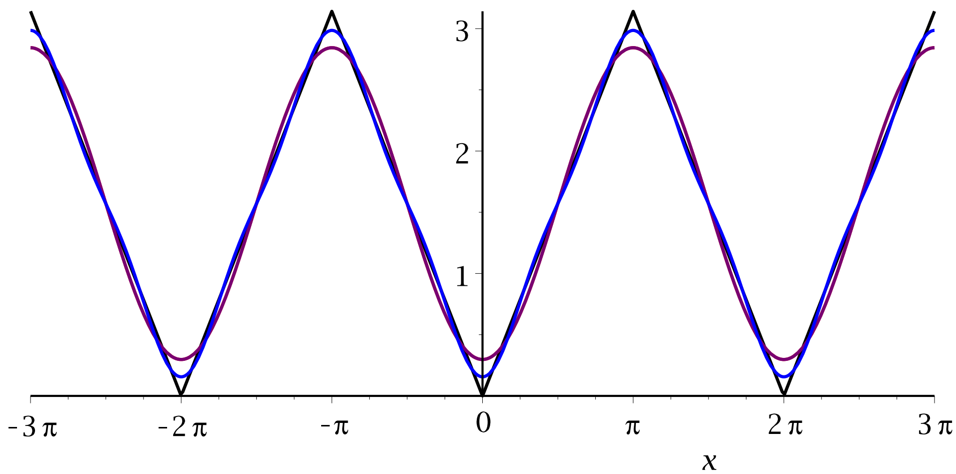
$$f := x \rightarrow piecewise(-\pi < x \textbf{ and } x < \pi, |x|, -3\pi < x \textbf{ and } x < -\pi, |x + 2\pi|, \pi < x \textbf{ and } x < 3\pi, |x - 2\pi|) \quad \textbf{(1)}$$

> $Ff := (x, nmax) \rightarrow \frac{\text{Pi}}{2} + sum\left(\frac{2\cdot((-1)^n - 1)}{\text{Pi}\cdot n^2}\cdot\cos(n\cdot x), n = 1..nmax\right);$

$$Ff := (x, nmax) \rightarrow \frac{1}{2}\pi + \sum_{n=1}^{nmax} \frac{(2(-1)^n - 2)\cos(n x)}{\pi n^2} \quad \textbf{(2)}$$

Below, I plot the function *f(x)* and its Fourier series to make sure that I have computed the Fourier coefficients correctly. Notice that the Fourier series appears to be converging uniformly to *f(x).*

> $plot([f(x), Ff(x, 1), Ff(x, 2), Ff(x, 3)], x = -3\cdot\text{Pi}..3\cdot\text{Pi}, colour = [black, red, purple, blue]);$



From the plot above, I notice that the error is largest at the points where the derivative is discontinuous, so I can compute the error at just one of these points for increasing values of *nmax*. I choose the point *x=0,* as that is the simplest one. Below I define the error function at the origin and plot it for increasing values of *nmax*. Since the error function is only defined for integer values of *nmax*, I need to make a listplot.

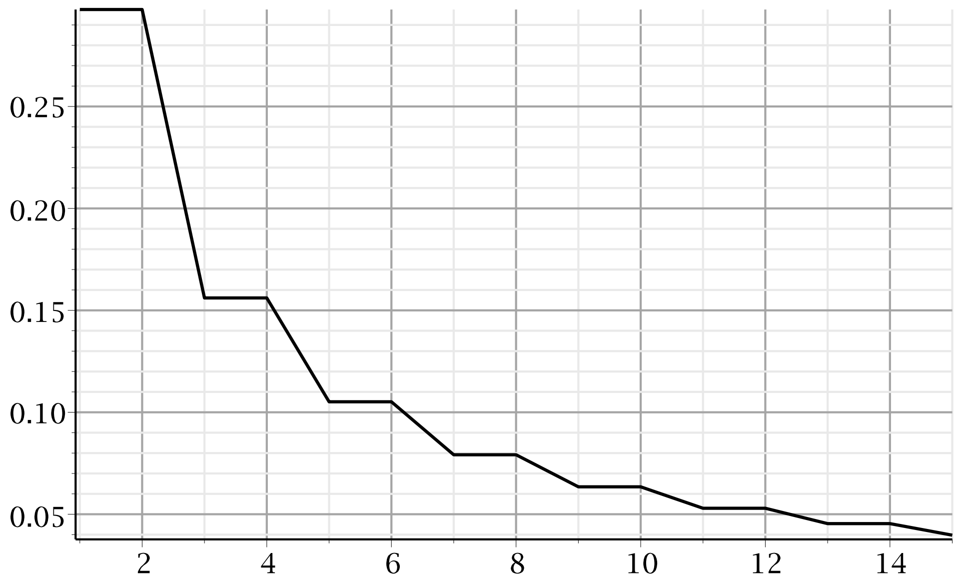
> $Ef := nmax \rightarrow evalf(\text{abs}(Ff(0, nmax) - f(0)));$

(3)

$$Ef := nmax \rightarrow evalf(|Ff(0, nmax) - f(0)|) \quad \textbf{(3)}$$

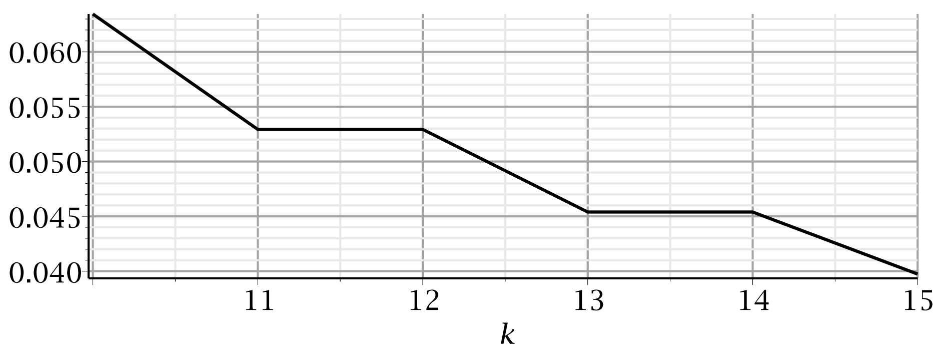
> $with(plots):$

> $listplot([seq([nmax, Ef(nmax)], nmax = 1..15)], gridlines = true);$



From the plot above, it looks as though the critical value of *nmax* is 13. Below I plot the error for *nmax*=10 to 15, so that we can see the crossing point of 0.05 more accurately.

> $\text{listplot}([\text{seq}([\text{nmax}, \text{Ef}(\text{nmax})], nmax = 10..15)], gridlines = true);$



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Now we can see clearly that if *nmax* is greater than or equal to 13, the error is below 0.05, as required.