Math 319 - Differential Equations II
Assignment # 5

Instructions: You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

1. Verify that the eigenfunctions of

$$\begin{aligned} &y'' + 2y' + (1+\lambda^2)y = 0, \qquad 0 < x < 1, \\ &y(0) = y'(1) = 0 \end{aligned}$$

   form an orthogonal set. Then express $f(x) = 1$ in an eigenfunction expansion.

2. Show that the problem

$$\begin{aligned} &x^2y'' - \lambda(xy' - y) = 0, \qquad 1 < x < 2, \\ &y(1) = 0, \qquad y(2) - y'(2) = 0, \end{aligned}$$

   has only one real eigenvalue, and find the corresponding eigenfunction.

3. Consider the following boundary value problem:

$$\begin{aligned} &x^2y'' + 2xy' + \frac{5}{4}y = h(x), \qquad 1 < x < e^{\pi}, \\ &y(1) = y\left(e^{\pi}\right) = 0. \end{aligned}$$

   (a) Find the adjoint boundary value problem for the associated homogeneous problem.
   (b) Determine the conditions on $h(x)$ that guarantee that the given nonhomogeneous boundary value problem has a solution.

4. Determine whether the given linear differential operator $L[y]$, whose domain consists of all functions that have continuous second derivatives on the interval $[0, \pi]$ and satisfy the given boundary conditions, is selfadjoint.

$$\begin{aligned} &L[y] = (1+x^3)y'' + 3x^2y' + \lambda r y, \\ &y'(0) = 0, \qquad y(\pi) - (1+\pi^3)y'(\pi) = 0. \end{aligned}$$