a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

Irving K. Barber School
of Arts and Sciences
UBC Okanagan

Instructor: Rebecca Tyson Course: MATH 319
Date: Nov 1st, 2023 Time: 12:30pm Duration: 50 minutes.
This exam has 4 questions for a total of 29 points.

UBC ID #: ____________________ NAME (print): ____

Signature: ____

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, ask for extra paper.

This is the individual portion of a two-stage exam.

1. Consider the PDE problem

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0, \qquad 0 < a < r < b, \quad -\pi < \theta < \pi, \tag{1a}$$

$$\frac{\partial u}{\partial r}(a,\theta) = 0, \quad u(b,\theta) = f(\theta), \qquad -\pi < \theta < \pi, \tag{1b}$$

[2] (a) Sketch the region on which the PDE problem (1) is defined, and label the boundaries according to (1b).

[4] (b) Set up *(do not solve!)* the two ODE problems that arise from using the separation of variables technique to solve the PDE problem. *Don't forget to specify the boundary conditions for one of the ODE problems!*

extra workspace for question 1

2. The Laplace equation on a disc of radius $r = 4$ leads to two ODE problems: The first is a BVP in θ,

$$T''(\theta) + \alpha^2 T(\theta) = 0, \quad -\pi < \theta < \pi, \tag{2a}$$

$$T(-\pi) = T(\pi), \quad T'(-\pi) = T'(\pi), \tag{2b}$$

and the second is a Cauchy-Euler ODE in r,

$$r^2 R''(r) + rR'(r) - \alpha^2 R(r) = 0, \quad 0 < r < 4. \tag{3}$$

Assume $\alpha > 0$.

5 (a) Solve the BVP (2).

4 (b) Solve the ODE (3). *Note the domain!!*

3. Consider the PDE problem

$$\frac{\partial^2 u}{\partial t^2} = \beta \frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 2\pi, \quad t > 0, \tag{4a}$$

$$\frac{\partial u}{\partial x}(0,t) = 0, \quad u(2\pi,t) = 0, \qquad t > 0, \tag{4b}$$

$$u(x,0) = f(x), \quad \frac{\partial u}{\partial t}(x,0) = g(x), \qquad 0 < x < 2\pi, \tag{4c}$$

where $\beta > 0$. By assuming $u(x,t) = X(x)T(t)$, this PDE becomes two ODE problems:

$$X'' + \lambda X = 0, \quad X'(0) = 0, \quad X(2\pi) = 0 \tag{5}$$

$$T'' + \lambda \beta T = 0 \tag{6}$$

1 (a) Why do the two initial conditions not appear in the ODE problem for $T(t)$?

7 (b) Assuming that $\lambda = w^2$, find a formal solution for $u(x,t)$.

extra workspace for question 3

6 4. Use Separation of Variables to reduce the PDE problem below to three ODE problems (do not solve the ODE problems, just set them up).

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}, \qquad 0 < x < a, \quad 0 < y < b, \quad t > 0, \tag{7a}$$

$$\frac{\partial u}{\partial x}(0, y, t) = \frac{\partial u}{\partial x}(a, y, t) = 0, \qquad 0 < y < b, \quad t > 0, \tag{7b}$$

$$u(x, 0, t) = U_1, \quad u(x, b, t) = U_2, \qquad 0 < x < a, \quad t > 0, \tag{7c}$$

$$u(x, y, 0) = g(x, y), \qquad 0 < x < a, \quad 0 < y < b. \tag{7d}$$

Include boundary conditions in the ODE problems where appropriate.

Question:	1	2	3	4	Total
Points:	6	9	8	6	29
Score:					