

Math 339 - Dynamical Systems
Assignment # 1
due Wed September 16th, 12:30pm

Instructions: You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

1. Exercise 1.1 (page 36)
2. Consider the map $f(x) = a(x^2 - x^3)$. This is the logistic map with an Allee effect. You can look up “Allee effect” if you’re interested.
 - (a) Plot $f(x)$ for various values of a between 1 and 6.75. Sketch the three representative cases.
 - (b) Analytically determine the steady states.
 - (c) Determine the stability of the zero steady state for arbitrary a .
 - (d) Determine the stability of the other two steady states for $a = 9/2$ (use the fraction!).
 - (e) Confirm your stability calculations by cobwebbing.

3. Consider the map

$$x_{n+1} = g(x_n) = 1 - a * x_n^2(1 - x_n). \tag{1}$$

- (a) Plot the map for $a = 5$. Indicate the steady states on your plot.
- (b) Predict what sort of bifurcation behaviour you should observe as a varies between 4 and 5. Back up your answer with plots of the map.
- (c) Create the bifurcation diagram for the map for $a = 4$ to 6.5. Use $x_0 = 0.5$ as your starting value. Is your diagram consistent with your prediction?