Math 339 - Dynamical Systems
Assignment # 2
due Wed September 23rd, 12:30pm

Instructions: You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

1. Consider the nonlinear map
$$f(x, y) = \left(2 \sin \left(\frac{\pi}{3} x \right), \frac{y}{2} \right).$$

   (a) Find all fixed points (you will need to use graphical techniques to find some of them - approximate values are acceptable for these).

   (b) By graphically iterating through the map a small disk centred around each fixed point, determine the stability of each fixed point.

2. Consider the linear map $f(\vec{x}) = A\vec{x}$, where the matrix A is the map. For each of the maps A below,

   (i) determine the stability of the fixed point at the origin (i.e. classify the fixed point as a source, sink or saddle),

   (ii) plot the image ellipse of the unit circle under two iterations of each map. Describe what are the dilation and rotation operations of the map in each case.

   $$\text{(a)} \quad A = \begin{bmatrix} 2 & 0.5 \\ 2 & -0.5 \end{bmatrix} \qquad \text{(b)} \quad A = \begin{bmatrix} 2 & 1 \\ -2 & 2 \end{bmatrix}$$

3. Exercise 2.1 (p 98)

4. Find and plot the stable and unstable manifolds of the saddle in the previous problem. Sketch typical trajectories for initial conditions in the four quadrants of the plane defined by the manifolds.