Math 339 - Dynamical Systems
Assignment # 3
due Wed September 30th, 12:30pm

Instructions: You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

1. Find all the fixed points of the three-dimensional map

$$f(x, y, z) = \left(x^2 y, y^4, \frac{xz}{2} + y\right),$$

and determine their stability.

2. Consider the map

$$f(x, y) = (ax - bx^2, x^2 + 2y),$$

where a and b are real numbers.

(a) For $a = 4$ and $b = 1$, verify that

$$\left\{ \left(\frac{5+\sqrt{5}}{2}, -\frac{15}{2} - \frac{5\sqrt{5}}{6} \right), \left(\frac{5-\sqrt{5}}{2}, -\frac{15}{2} + \frac{5\sqrt{5}}{6} \right) \right\}$$

is a period-2 orbit, and determine its stability.

(b) For $a = 1/3$ and $b = 0$, show that $(0, 0)$ is a saddle fixed point.

3. Consider the map $f(x, y) = (ax, bx^3 + cy)$.

(a) Show that (0,0) is a saddle fixed point for either $|a| < 1$ & $|c| > 1$ or $|a| > 1$ & $|c| < 1$.

(b) Find the inverse map f^{-1}.

(c) Show that the set $S = \{(t, qt^3) : t \in \Re\}$, where $q = b/(a^3 - c)$ is invariant.

(d) Determine the stable and unstable manifolds for each case in question 3a.