# Math 339 - Dynamical Systems
# Assignment # 4
# due Wed October 14th, 12:30pm

**Instructions:** You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

1. Consider the first-order equation

$$\frac{dn}{dt} = rn\left(1 - \frac{n}{q}\right) - \frac{n^2}{1+n^2}.$$

   This model applies to a population $n$ that grows logistically, with intrinsic growth rate $r$ and carrying capacity $q$, and that suffers predation by a constant level of generalist predators. It was used to explain the regular periodic outbreaks of the spruce budworm in Eastern Canada[1]. In the following, assume $q > 6$.

   (a) Find the fixed points using graphical and analytic techniques, as appropriate. Show that (1) can have 1, 2, or 3 fixed points as $r$ varies. Sketch the phase line in each case.

   (b) Classify each fixed point as stable or unstable (sink or source).

   (c) Sketch a bifurcation diagram with fixed point coordinates on the vertical axis, and $r$ on the horizontal axis. Show both unstable and stable fixed points, using dashed lines to represent the coordinates of unstable fixed points, and solid lines to represent the coordinates of stable fixed points.

2. Find the general solution of

$$2y'' - 7y' + 3y = 0.$$

3. Solve the initial value problem

$$y'' + 2y' + y = 0, \qquad y(0) = 5, \quad y'(0) = -3.$$

4. Find the general solution of

$$y'' - 4y' + 5y = 0.$$

5. Write the third order equation below as a system of three first order equations in normal form.

$$x''' - 6x' + (8 - x)x = 0.$$

[1] D. Ludwig, D.D. Jones, and C.S. Holling (1978) Qualitative analysis of insect outbreak systems: The spruce budworm and forest. *Journal of Animal Ecology* **47**:315-332