Math 339 - Dynamical Systems
Assignment # 5
due Wed October 21st, 12:30pm

Instructions: You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

1. Consider the second-order equation $\ddot{x} + 3\dot{x} - 4x = 0$. It can be written as a system of two first order equations in normal form by introducing two new variables, $u_1 = x$ and $u_2 = \dot{x}$. The equivalent system is

$$\frac{du_1}{dt} = u_2, \tag{1a}$$
$$\frac{du_2}{dt} = -3u_2 + 4u_1. \tag{1b}$$

   (a) Verify that the linear system is indeed equivalent to the second order ODE.

   (b) Use the eigenvalues and eigenvectors of the coefficient matrix of (1) to determine the phase plane solution $\vec{u} = (u_1, u_2)$.

   (c) Sketch a few solutions in the phase plane.

2. Consider the system

$$\frac{dx}{dt} = 2x - y \tag{2}$$
$$\frac{dy}{dt} = x^2 + 4y. \tag{3}$$

   Find all equilibria and classify them as asumptotically stable, stable, or unstable. Verify your results by producing a phase plane plot with Maple.

3. Exercise T7.7 (p 296).

4. A pair of competing species can be described mathematically as interacting according to the equations

$$\dot{x} = 3x(1-x) - xy, \tag{4a}$$
$$\dot{y} = 5y(1-y) - 2xy. \tag{4b}$$

   Each species grows logistically ($3x(1-x)$ in (4a) and $5y(1-y)$ in (4b)), and then is predated by the other species according to a mass-action process (xy in (4a) and $2xy$ in (4b)). Note that species y has a higher intrinsic growth rate than species x ($5 > 3$), but that species x exerts more predation pressure on y than y does on x ($2xy > xy$). So, just from looking at the equations, it is not clear if one species will outcompete the other, or if they will be able to coexist. Your task is to determine which of these outcomes prevails.

(a) Sketch the nullclines. Note that we are only interested in the 1st quadrant where $x > 0$ and $y > 0$.

(b) Find the steady states.

(c) Determine the stability of each steady state.

(d) Use maple to plot the phase plane and a few solutions. Explain how your phase plane is consistent with your results in part 4c.

(e) Explain, using all of your analytic and numeric (maple) work, whether the outcome is coexistence of both species, or extinction of one.