INSTRUCTOR: Rebecca Tyson

COURSE: MATH 339

Irving K. Barber School
of Arts and Sciences
UBC Okanagan

Date: Oct 2nd, 2015 Time: 12:00pm Duration 45 minutes.
This exam has 5 questions for a total of 28 points.

SPECIAL INSTRUCTIONS

- Show and explain all of your work unless the question directs otherwise. Simplify all answers.
- The use of a calculator is permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Problem	Points Earned	Points Out Of
1		5
2		5
3		4
4		3
5		5
TOTAL:		**28**

CANDIDATE NAME (print): ______________________

STUDENT NUMBER: ______________________

Signature: ______________________

$\boxed{5}$ 1. Consider the map $f(x)$ plotted, with the identity line, in Figure 1. Indicate all of the steady states on the plot, and then determine the stability of each steady state using graphical techniques. Explain how you arrive at your stability results.

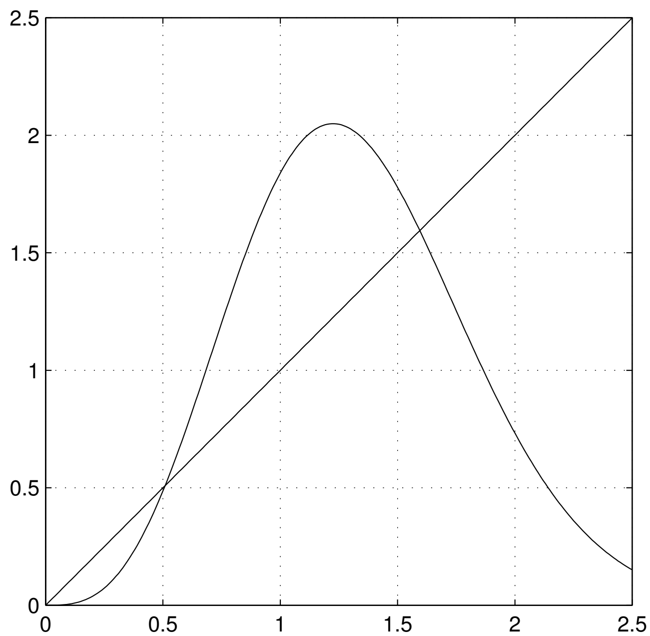
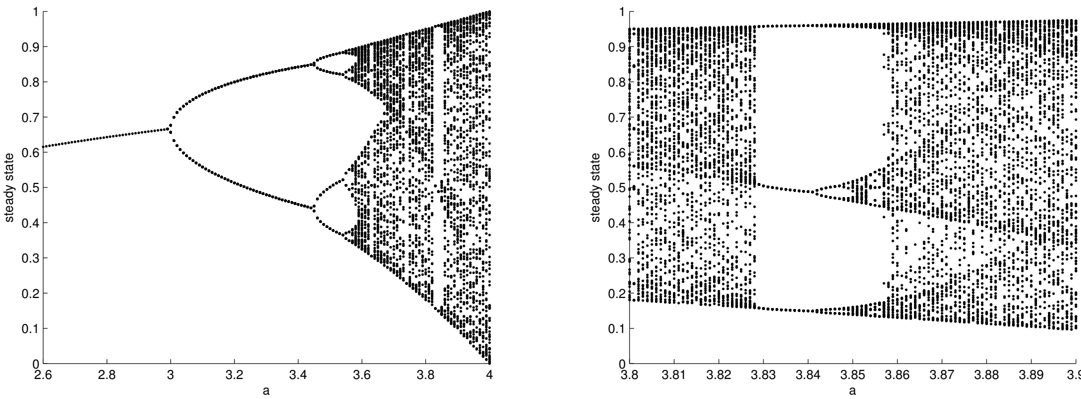


Figure 1: Map for problem 1.

$\boxed{5}$ 2. Below are two plots showing parts of the bifurcation diagram for the logistic map, $f(x) = ax(1-x)$. The right panel is a zoomed-in view of part of the left panel.

(a) For each range of a values given below, describe what is the behaviour of the system (i.e. what is the period of the stable orbit, any other special names for the behaviour or sequence of behavours).



$\boxed{1}$ i. $2.6 < a < 3.5$

$\boxed{1}$ ii. $3.83 < a < 3.85$

$\boxed{1}$ iii. $a = 4$

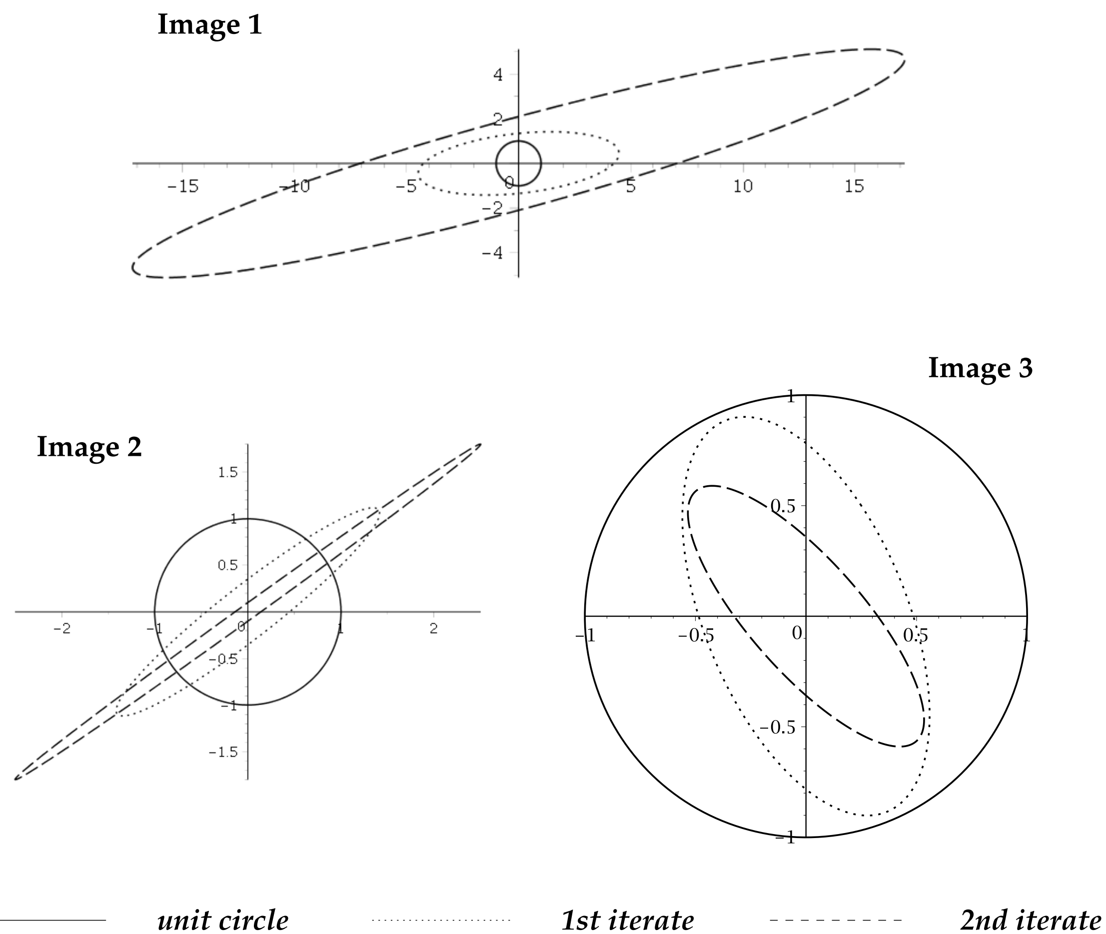
$\boxed{2}$ (b) At $a = 3$, the left panel shows a bifurcation in model behaviour. Mathematically, what happens to the stability of the fixed point x^* of f as a increases past 3? Each point in the period-2 orbit that emerges is a fixed point of what map?

$\boxed{4}$ 3. Consider the linear map $f(\vec{x}) = A\vec{x}$, where

$$A = \begin{bmatrix} 1/4 & -1/2 \\ 1/2 & 3/4 \end{bmatrix}.$$

Determine the stability of the steady state at the origin.

$\boxed{4}$ 4. Below are three plots showing the image of the unit circle under two iterations of three different linear maps. For each plot, classify the steady state at the origin. Explain your answer in each case.



5. Consider the saddle fixed point $\vec{0}$ of the map $f(x, y) = (x/2, 2y - 7x^2)$, and the following information:

   - The inverse map is $f^{-1}(x, y) = (2x, y/2 + 14x^2)$.
   - The y axis is invariant under f, and $|f^{-n}(0, y) \to 0|$ as $n \to \infty$.

   With this information, answer the questions below.

$\boxed{3}$ (a) Show that the set $S = \{(x, 4x^2) : x \in \mathbb{R}\}$ is invariant under f, and that $|f^n(x, 4x^2) \to 0|$ as $n \to \infty$.

$\boxed{2}$ (b) Sketch the stable and unstable manifolds of $\vec{0}$, and some nearby trajectories.