Math 339 - Dynamical Systems
Assignment # 2
due Fri September 22nd, 4pm

Instructions: You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

1. (taken from Exercise 1.12) Consider the map $h(x) = axe^{-x}$.
   (a) Find the fixed points of the map $h(x)$.
   (b) Find out for what value of a the positive steady state x^* is a **superstable fixed point** x^*, that is, $h'(x^*) = 0$. Denote by a_{ss} this value of a.
   (c) Investigate the orbit starting at $x_0 = 0.1$ for $a = a_{ss}$ using Maple. How does the behaviour of the orbit change if a is increased by 50%?
   (d) What is the range of $a \geq 1$ for which the steady state x^* is a sink?
   (e) As a increases through the range identified above, it eventually reaches a critical value a_c above which the orbits approach a two-cycles. Verify this result. Plot the second order map and identify the steady states associated with the fixed point and with the two-cycle.
   (f) Use Maple to explore the behaviour of solutions as a increases above a_c. Use Maple to produce a bifurcation plot. Explain what you observe.
2. Exercise 1.14
3. Exercise 2.5